Linear Algebra   Math 3A

Final Exam

Monday, March 20, 2017 — 4:00 pm - 6:00 pm

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

Find a basis in $Col(A)$, where

$$A = \begin{pmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & -1 & 3 \\ 3 & 6 & 2 & 22 \\ 4 & 8 & 0 & 16 \end{pmatrix}.$$

Answer: One can take the first and the third columns of the matrix as a basis; notice that the choice of a basis is not unique.

Problem 2.

Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$, where

$$A = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix}.$$

Answer: $P = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.

Problem 3.

For each of the following statements determine whether it is true or false. Explain your answers.

a) If A is 3×3 matrix that has three distinct real eigenvalues, then A^{2017} also has three distinct real eigenvalues.

b) For any 2×2 matrix A one has rank $A =$ rank A^2.

c) If A and B are similar matrices, then A^3 and B^3 are also similar.

Answer: a) true; b) false; c) true. Notice that the answers had to be explained!

Problem 4.

Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & t \\ 1 & 4 & t^2 \end{pmatrix}.$$

For what values of t is A invertible?

Answer: for every t that is not equal to 1 or 2.

Problem 5.

Suppose $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ is a linearly independent set of vectors from $\mathbb{R}^{2017}$. Does it imply that the set of vectors

$$\{2\bar{v}_1 + 3\bar{v}_2 + \bar{v}_3,\ \bar{v}_1 - \bar{v}_2 + 2\bar{v}_3,\ 2\bar{v}_1 + \bar{v}_2 - \bar{v}_3\}$$

is linearly independent?

Answer: yes.