Math 3A, Linear Algebra
Practice problems

Problem 1.

Is the solution set of a system of 3 homogeneous equations in 5 variables a subspace in $\mathbb{R}^3$ or in $\mathbb{R}^5$?

Problem 2.

True or False: If A and B are $n \times n$ matrices, then $(A+B)(A-B) = A^2 - B^2$.

Problem 3.

Suppose $A^n = 0$ for some $n > 1$. Find an inverse for $I - A$.

Problem 4.

True or False: If $\bar{u} \perp \bar{v}$, then $\|\bar{u} - \bar{v}\| = \|\bar{u} + \bar{v}\|$.

Problem 5.

Let x_1, x_2, x_3 be real numbers. Define Vandermond matrix V by

$$V = \begin{pmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ x_1^2 & x_2^2 & x_3^2 \end{pmatrix}.$$

Show that $\det V \neq 0$ if and only if all the numbers x_1, x_2, x_3 are different.

Problem 6.

True or False: If $\bar{v}$ and $\bar{u}$ are vectors from $\mathbb{R}^3$, and $\bar{v} \perp \bar{u}$, then for any vector $\bar{x}$ from $\mathbb{R}^3$ we have

$$\|\bar{x}\|^2 \geq |\bar{x} \cdot \bar{v}|^2 + |\bar{x} \cdot \bar{u}|^2.$$

Problem 7.

True or False: If matrix A is similar to matrix B, and B is diagonalizable, then A is also diagonalizable.

Problem 8.

Suppose an $n \times n$ matrix A satisfies the equation $A^2 - 2A + I = 0$. Show that $A^3 = 3A - 2I$.