

MATH 3A, LINEAR ALGEBRA

SAMPLE FINAL

Problem 1.

Find a basis in the subspace spanned by vectors

$$\begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ -7 \end{bmatrix}.$$

Problem 2.

Find $\det A$, where

$$A = \begin{pmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{pmatrix}.$$

Problem 3.

Find all eigenvalues of the matrix

$$\begin{pmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

Problem 4.

For each of the following statements determine whether it is true or false (explain your answer):

- If A and B are diagonalizable 2×2 matrices, then $A + B$ is also diagonalizable.
- If A , B , and C are 2×2 matrices, such that A and B are similar, and B and C are similar, then A and C are also similar.
- If A and B are 2×2 matrices that have no real eigenvalues, then AB also does not have real eigenvalues.

Problem 5.

Find A^n (for every $n = 2, 3, \dots$), where

$$A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}.$$