MATH 3A, LINEAR ALGEBRA
SAMPLE FINAL

Problem 1.

Find a basis in the subspace spanned by vectors

$$\begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ -7 \end{bmatrix}.$$

Answer: For example, vectors $\begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \\ -1 \end{bmatrix}$ form a basis. Notice that the choice of a basis is not unique!

Problem 2.

Find det A, where

$$A = \begin{pmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{pmatrix}.$$

Answer: -10

Problem 3.

Find all eigenvalues of the matrix

$$\begin{pmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

Answer: $-1, 4, 2$.

Problem 4.

For each of the following statements determine whether it is true or false (explain your answer):

a) If A and B are diagonalizable 2×2 matrices, then $A + B$ is also diagnalizable.

b) If A, B, and C are 2×2 matrices, such that A and B are similar, and B and C are similar, then A and C are also similar.

c) If A and B are 2×2 matrices that have no real eigenvalues, then AB also does not have real eigenvalues.

Answer: (a) - false; counterexample: $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$.

(b) - true; indeed, if $A = PBP^{-1}$ and $B = QCQ^{-1}$, then $A = (PQ)C(PQ)^{-1}$.

(c) - false; counterexample: $A = B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

Problem 5.

Find A^n (for every $n = 2, 3, \ldots$), where

$$A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}.$$

Answer: $A^n = \begin{pmatrix} 2 \cdot 3^n - 2^n & 2 \cdot 3^n - 2^{n+1} \\ 2^n - 3^n & 2^{n+1} - 3^n \end{pmatrix}$.