MATH 3A, LINEAR ALGEBRA
SAMPLE MIDTERM

Problem 1.

Solve the system of linear equations

$$\begin{cases} x_1 + x_2 - 2x_3 &= 1 \\ 2x_1 - 3x_2 + x_3 &= -8 \\ 3x_1 + x_2 + 4x_3 &= 7 \end{cases}$$

Problem 2.

For which values of the parameter h the vectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, and $\mathbf{v}_3 = \begin{pmatrix} 1 \\ 0 \\ h \end{pmatrix}$, are linearly dependent?

Problem 3.

Which of the following transformations are linear? For those that are linear, find the standard matrix. For those that are not, explain why they are not linear.

a) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x_1, x_2) = (x_2, x_1 - 2x_2)$;

b) $T : \mathbb{R}^3 \to \mathbb{R}^1$, $T(x_1, x_2, x_3) = |x_1 + x_2 + x_3|$;

c) $T : \mathbb{R}^1 \to \mathbb{R}^3$, $T(x_1) = (x_1, x_1^2, x_1^3)$.

Problem 4.

Find A^8, where $A = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$.

Problem 5.

Find A^{-1}, where

a) $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$,

b) $A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$.