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Tian, Jianjun Paul (1-OHS-IMB); **Vojtěchovský, Petr** (1-DNV-MD)
Mathematical concepts of evolution algebras in non-Mendelian genetics. (English summary)
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An evolution algebra is a finite-dimensional algebra E with basis $\{e_1,\dots,e_n\}$ and multiplication given by $e_ie_i=\sum_{j=1}^n a_{ij}e_j$ and $e_ie_j=0$ $(i\neq j)$. Such an algebra describes self-reproduction of non-Mendelian genetics. The elements $e_1,\dots,e_n$ represent alleles and a_{ij} is the probability that e_i becomes e_j in the next generation.

The authors obtain elementary algebraic properties of nondegenerate evolution algebras (i.e., $e_ie_i\neq 0$ for every $i=1,\dots,n$) over arbitrary fields. Evolution algebras are commutative (and hence flexible) but are not necessarily power-associative. Not every evolution algebra is a Banach algebra. However, if E is a real evolution algebra such that $\sum_{j=1}^n |a_{ij}|\leq 1$ for every $i=1,\dots,n$, then E is a Banach algebra. Evolution algebras are not closed under subalgebras. A subalgebra F of E is called an evolution subalgebra if $F=\{\sum_{i=1}^n \alpha_ie_i|\alpha_i=0,\forall i\notin I\}$ for some $I\subset\{1,\dots,n\}$. An evolution algebra is evolutionary simple if it has no proper evolution subalgebras. An evolution algebra is evolutionary semisimple if it is a direct sum of some of its evolutionary simple evolution subalgebras. Every evolution algebra E decomposes as a vector space into $P(E)\oplus T(E)$, where $P(E)$ is an evolutionary semisimple evolution algebra and $T(E)$ is a subspace (not necessarily a subalgebra) of E that can be made into an evolution algebra. As stated by the authors, the algebraic properties are of importance from the biological point of view.

Luiz Antonio Peresi