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Sur les algèbres de Bernstein. **(French)** **[On Bernstein algebras]**
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A Bernstein algebra A over a commutative ring K is one that admits a homomorphism $w\colon A \to K$ and satisfies the identity $(x^2)^2 = \{w(x)\}^2 x^2$, $x \in A$. Relative to each idempotent e, $\operatorname{Ker} w$ has a Pierce decomposition $U \oplus V$, such that $ex = \frac{1}{2}x$ for $x \in U$, $ex = 0$ for $x \in V$. Jacobi's identity holds in U, $x(yz) + y(xz) = 0$ for $x, y \in U$, $z \in V$, $(xy)(zt) + (xz)(yt) + (xt)(yz) = 0$ for $x, y, z \in \operatorname{Ker} w$, $t \in A$, and $U^2 \subset V$, $UV \subset U$, $V^2 \in U$, $UV^2 = 0$.

The authors begin with the study of derivations of Bernstein algebras. Theorem 3.1 gives the necessary and sufficient conditions for a linear mapping d to be a derivation when $\operatorname{char} K \neq 2$. They include $d(e) \in U$ and the fact that d has the representations $d(x) = f_d(x) + 2xd(e)$, $d(x) = -2xd(e) + g_d(x)$ where f_d, g_d are endomorphisms of U, V, respectively, corresponding to d in a morphism of Lie algebras $f\colon \operatorname{Der}_K(A) \to \operatorname{End}_K(U)$, $g\colon \operatorname{Der}_K(A) \to \operatorname{End}_K(V)$. The remaining conditions relate to the effects of f and g on U, V. As a corollary we have $w \cdot d = 0$. The type of a Bernstein algebra is $(\dim U + 1, \dim V)$. The theorem is applied to Bernstein algebras of the extreme types. If $\dim A = n + 1$, type $A = (n + 1, 0)$, then $\operatorname{Der}_K(A)$ is isomorphic to $K^n \times M_n(K)$, the product being semidirect and M_n the full matrix algebra over K. If type $A = (1, n)$, then $\operatorname{Der}_K(A)$ is isomorphic to $M_n(K)$.

The next section deals with derivations in the case $\operatorname{char} K = 2$. Here, the conditions include $d(e) = 0$, a decomposition $d(x) = (w \cdot d)(x)e + f_d(x)$, and an appropriately modified set of detailed identities. In order that $w \cdot d = 0$, it is necessary and sufficient that f should be injective. Further interesting results are obtained for $\operatorname{char} K = 2$, the first time that this case has been extensively studied in the context of genetic algebras. The next sections deal with the automorphism group of A for $\operatorname{char} K \neq 2$, $= 2$, respectively. The general theorems are, mutatis mutandis, related to those on derivations, but the exposition is illustrated by a wide range of examples. An important role is played by the abelian group $K(\theta)$, $\theta \in K$. It comprises those elements $\lambda \in K$ such that $1 - 4\lambda\theta \in U(K)$, U the group of invertible elements of K (an unfortunate clash of notation), with addition defined by $\lambda \oplus \lambda' = \lambda + \lambda' - 4\lambda\lambda'$. In some cases $\operatorname{Aut}^K(A)$ is isomorphic to $K(\theta)$. If type $A = (n + 1, 0)$, then $\operatorname{Aut}_K(A)$ is isomorphic to $I_p(A) \times \operatorname{GL}_K(U)$, while if type $A = (1, n)$, $\operatorname{Aut}_K(A)$ is isomorphic to $\operatorname{GL}_K(V)$. The final section examines in detail the cases of all Bernstein algebras of dimension 3.

{See also the following review.} *P. Holgate*