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González, S. [González Jiménez, Santos] (E-UO); **López-Díaz, M. C.** (E-UO);
Martínez, C. [Martínez López, Consuelo] (E-UO);
Shestakov, Ivan P. (RS-AOSSI)

Bernstein superalgebras and supermodules. (English summary)

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In this paper the authors define (U,W)-graded Bernstein algebras. Initially they prove that if (B,ω) is a Bernstein algebra then $N(B)=\ker\omega$ is a (U_e,V_e)-graded Bernstein algebra, where $e\in A$ is a nonzero idempotent and U_e and V_e are the subspaces of the Peirce decomposition of B relative to e. Conversely if A is a (U,W)-graded Bernstein algebra then the extended algebra $A(e)=Fe+U+W$ is a Bernstein algebra.

The concept of isotopy is introduced for (U,W)-graded Bernstein algebras and it is proved that two graded Bernstein algebras are isotopic if and only if their extended algebras are isomorphic.

In this context, it is possible to define Bernstein superalgebra and it is proved that there are no semiprime Bernstein superalgebras over a field F of characteristic $\neq 2,3$.

Following this line the authors define (X,Y)-Bernstein supermodule over a (U,W)-graded Bernstein algebra. They give two examples of irreducible Bernstein supermodules, supermodules of type $M(K,V,\alpha)$ and type $M(W,X)$. They prove that if A is a (U,W)-Bernstein superalgebra over a field F of $\operatorname{char} F\neq 2,3$ and M is an almost faithful irreducible (X,Y)-Bernstein supermodule over A then M is isomorphic to a supermodule of one of the types: $M(K,V,\alpha)$, $M(K,V,\alpha)^s$, $M(W,X)$, where an A-(super)module M is almost faithful if its annihilator $\operatorname{Ann} M=\{a\in A|\ Ma=0\}$ does not contain nonzero ideals of A.

Henrique Guzzo, Jr.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.