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Mallol, Cristián (RCH-FRN-EM); **Varro, Richard** (F-MONT3-MA); **Benavides, Raúl** (RCH-FRN-MS)
Gamétisation d'algèbres pondérées. **(French. French summary)** **[Gametization of weighted algebras]**
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The paper studies the notion of gametization of baric (or weighted) algebras. A baric algebra A over a field F, $\operatorname{ch} F \neq 2$ (there is a nonzero homomorphism $\omega\colon A \to F$) is a gametic algebra if it satisfies $x^2 = \omega(x)x\ \forall x \in A$.

If (A,ω) is a baric algebra, the new product $x \cdot y = \frac{1}{2}(\omega(y)x + \omega(x)y)$ defines a new structure of gametic algebra on A.

If $\gamma \in K$ and (A,ω) is a baric algebra, the gametization of A acording to γ, A_γ, is the algebra with the same underlying vector space of A and the new product

$$(xy)_\gamma = (1-\gamma)xy + \frac{1}{2}\gamma(\omega(y)x + \omega(x)y).$$

The following theorem is proved in the paper: Theorem. Let (A,ω) be a baric algebra and $\gamma \in K - \{1\}$. Then (a) $\operatorname{Aut}(A_\gamma) = \operatorname{Aut}(A)$, (b) $\operatorname{Der}(A_\gamma) = \operatorname{Der}(A)$, (c) $\operatorname{Nilp}(A_\gamma) = \operatorname{Nilp}(A)$, (d) A_γ and A have the same idempotent elements of weight equal to 1, (e) an idempotent E gives the same Peirce decomposition in A and in A_γ.

The reason given by the authors to introduce the gametization of a baric algebra is the study of the structures that satisfy ω-polynomial identities.

The authors prove that if A satisfies an ω-polynomial identity, then A_γ satisfies another identity with the same leading monomial.

In particular, the gametization of a train identity is another train identity with the same degree and it is proved that for any $n \geq 2$ there is exactly one train identity (the universal invariant) that is invariant under every gametization.

In this way the authors can identify all train identities of degrees 3 and 4 and those having $(x^2)^2$ as leading monomial. *Consuelo Martínez*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.