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Sur les algèbres de Bernstein. IV. (French. English summary) [On Bernstein algebras. IV]
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A commutative algebra A is nth order Bernstein if it admits a homomorphism $x \to w(x)$ into its coefficient field K, and satisfies the identity $x^{[n+2]} = (w(x)x)^{[n+1]}$, where $x^{[n]}$ is the nth plenary power, defined by $x^{[1]} = x$, $x^{[n+1]} = (x^{[n]})^2$. The name Bernstein algebra was introduced before for the case $n = 1$, which has been widely studied.

The authors show that in characteristic 2, an algebra is nth order Bernstein if and only if it contains an ideal I with $A/I \approx K$ and $x^{[n+1]} = 0$ for all $x \in I$. They obtain a number of other results for order 1, char $K = 2$. For example, the condition $e(ex) = ex$, $(ex)x = 0$ for fixed idempotent e, all x, is equivalent to power associativity.

In char $K \neq 2$ the authors study the decomposition of the vector space relative to the powers of multiplication by a fixed idempotent. They introduce the idea of coherence, which means roughly that the differences of the null spaces of the powers of e are 1-dimensional and exhaust the whole space. The associative algebra generated by the above multiplications is called the associated algebra $\mathrm{Ber}_K(A, w)$ and it and its subalgebras are studied. A further section develops the presentation of the structure of A via the equation $(e + \mu x)^{[n+1]} = e + \sum \mu^k \Lambda_{k,n}$. The use of the $\Lambda_{k,n}$ simplifies the derivation of useful identities, which are obtained. This leads on to a series of results that explore the generalisations of relations between Bernstein algebras and quasiconstant algebras to the nth order case. The paper then concentrates on $n = 2$. Many detailed results are given, of which "every Bernstein algebra of order 2, dimension 3 is quasiconstant of order 2" is typical. The final sections focus on the classification and listing of the possibilities for small-dimensional algebras of this class.

Part III has not been received by MR. Parts I and II have been reviewed [MR0969546 (90b:17047); MR0993027 (90d:17030)]. *P. Holgate*