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$G = G_A \oplus G_M$ is the Grassmann algebra) is (-1,1) algebra. Similarly as in [8] we have

**Corollary.** *A (-1,1) superalgebra* $B = A \oplus M$ *is solvable iff its even component A is solvable.*

**Remark.** An example of Shestakov ([7], Example 3) shows that in $Z_2$-graded (-1,1) algebras solvability does not imply nilpotency: $B = A \oplus M$, $B^{(2)} = 0$ (in particular $A^{(2)} = 0$), but $B$ is not nilpotent.

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**ALGEBRAS GROUPS AND GEOMETRIES 14, 31-39 (1997)** 

# DERIVATIONS IN SOME BERNSTEIN ALGEBRAS OF ORDER 2

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**Abstract**

The structure of the exceptional and power-associative Bernstein algebras of order 2 have recently being elucidated. In this paper we show that these two subsets of the Bernstein algebras of order 2 are distinct. We find necessary and sufficient conditions for a linear transformation to be a derivation for both of these classes of Bernstein algebras of order 2. Moreover, we prove some results related to both derivations and Peirce transformations.

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# 1 Introduction.

Let $A$ be a commutative not necessarily associative algebra over an infinite field $K$ with $\mathrm{char}(K) \neq 2$. If $w : A \to K$ is a nonzero algebra homomorphism, then the pair $(A, w)$ is called a *baric algebra* and $w$ its *weight homomorphism*. If the baric algebra $(A, w)$ satisfies the identity $x^{[n+2]} = (w(x)x)^{[n+1]}$, it is called a *Bernstein algebra of order* $n$, where $n$ is the minimum integer for which the identity holds and $x^{[1]} = x, \cdots, x^{[k+1]} = x^{[k]}x^{[k]}$, $k \geq 1$ are the plenary powers of $x$ (see [8] and [10] for references).

In a Bernstein algebra of order 2 an element of the form $x^{[3]} = w(x)^{-4}x^{[2]}$ with $w(x) \neq 0$ is an idempotent element. In the general case, not even the set of idempotent elements $I_p(A)$ of $A$ is known. In [10] Ouattara characterizes the set of generalized idempotents of a Bernstein algebra of order $n$, that is, the elements of $A$ such that $e = e^{[k+1]}$ for some $k \geq 2$. In [5] the authors characterize the set of idempotents of a Bernstein algebra of order 2 in some particular cases. In the general case they prove that the set $I = \{(e + u + u^2)^2 \ / \ u \in U\}$ is contained in the set $I_p(A)$.

Let $e \neq 0$ be an idempotent element and $L_e : Ker(w) \to Ker(w)$ the linear operator defined by $L_e(x) = ex$ for all $x \in Ker(w)$. $L_e$ satisfies $L_e^2(2L_e - I) = 0$. If we write $\mathrm{Ker}(w) = N$, we have a Peirce decomposition $A = Ke \oplus N = Ke \oplus U \oplus V_2$, where $U = Ker(2L_e - I)$ and $V_2 = Ker(L_e^2)$. Moreover, the subspaces $U$ and $V_2$ satisfy the relation $U^2 \subseteq V_2$ (see [8] and [10] for details).

*In the following let A be a Bernstein algebra of order 2.*

We say that $A$ is *exceptional* if $U^2 = \{0\}$. It is known that $U^2 = \{0\}$ is independent of the choice of the idempotent element.

It is also known[11] that in these algebras we have, for every $x \in \mathrm{Ker}(w)$

$$e(ex) = 2e(e(ex)). \tag{1}$$

Hence,



$$e(ex) \in U. \tag{2}$$

Relation (1) implies that for every $x \in \mathrm{Ker}(w)$ we have

$$x - 4e(ex) \in V_2. \tag{3}$$

Moreover, for every $u \in U$, $v \in V_2$ we have

$$u(ev) + e(uv) \in U. \tag{4}$$

Further information about exceptional Bernstein algebras of order 2 can be found in [11].

We say that the algebra $A$ is called a *power-associative algebra* if for all $x \in A$ the subalgebra generated by $x$ is associative.

If $A = Ke \oplus U \oplus V_2$ is the Peirce decomposition of a power-associative Bernstein algebra of order 2, then $eV_2 = \{0\}$, $UV_2 \subseteq U$, $V_2^2 \subseteq V_2$, $V_2^2 \neq \{0\}$ and for every $u$, $u' \in U$, $v$, $v' \in V_2$ we have:

$$u^3 = 0, \ v^4 = (v^2)^2 = 0, \tag{5}$$

$$u(vv') = (uv)v' + (uv')v, \tag{6}$$

$$v(uu') = (vu)u' + (vu')u. \tag{7}$$

(see [7] for details).

It is known that in Bernstein algebras of order 2, $\dim(U)$ and $\dim(V_2)$ are invariants of $A$, that is, these dimensions remain inchanged if $e$ is replaced by another idempotent element and the pair $(1+\dim(U), \dim(V_2))$ is an invariant of $A$, called *the type* of $A$.

If $A$ has type $(1+r, t)$ then $\dim(U^2) \leq \frac{1}{2}r(r+1)$, but also $\dim(U^2) \leq t$ because $U^2 \subseteq V_2$. Thus $\dim(U^2) \leq \min\{t, \frac{1}{2}r(r+1)\}$.

Let us consider the following two subsets of the Bernstein algebras of order 2:

$$B_{EX} = \{(A, w) | A \ \ is \ \ exceptional\}$$

and

$$B_P = \{(A,w) | A \ is \ power-associative\}.$$

First, we observe that $B_{EX} \neq B_P$, as the following examples show.

**Example 1.-** Let $A = Ke\oplus < u > \oplus < v_1, \cdots, v_n >$ be a commutative algebra with multiplication given by $e^2 = e$, $eu = ue = \frac{1}{2}u$, $ev_1 = v_1e = v_2$, $v_1^2 = v_2$, all other products being zero. Then $A$ is a Bernstein algebra of order 2 of type $(1+1,n)$. It is exceptional but not power- associative, since by taking $a = e + v_1$, we have $(a^2)^2 = e \neq a^4 = e + v_2$.

**Example 2.-** Let $A = Ke\oplus < u_1, u_2 > \oplus < v_1, \cdots, v_n >$ be a commutative algebra with multiplication given by $e^2 = e$, $eu_i = u_ie = \frac{1}{2}u_i$, $(i = 1,2)$, $u_1^2 = v_2$, $v_1^2 = v_2$, $u_1v_1 = u_2$, all other products being zero. Then the elements of $A$ satisfy the identity $(x^2)^2 = w(x)x^3$ that charaterizes a power-associative Bernstein algebra of order 2 with $v^3 = 0 \ \forall v \in V_2$ (see [2], Theorem 2.3). Thus $A$ is an element of $B_P$, but $A \notin B_{EX}$.

**Remark 1.** We can observe that even in the case $U = \{0\}$, there do exist exceptional Bernstein algebras of order 2 which are not power-associative. For example, the algebra $A = Ke\oplus < v_1, \cdots, v_n >$ with multiplication table: $e^2 = e$, $ev_1 = v_1e = v_2$, $v_1^2 = v_2$, all other products being zero is an element of $B_{EX}$ and $A \notin B_P$ since $((e+v_1)^2)^2 = e \neq (e+v_1)^4 = e+v_2$.

## 2 Derivations

The derivation algebras associated to various non-necessarily associative algebras are well known (e.g. the gametic algebra, the zigotic algebra, Bernstein algebra of order 1, etc). Some interpretations of derivations in genetic algebras are also known (see [6]).

In the following, let $A = Ke \oplus U \oplus V_2$ be a Bernstein algebra of order 2.

A derivation $D$ on $A$ is a linear mapping of $A$ into $A$ such that $D(xy) = D(x)y + xD(y) \quad \forall x, y \in A$ (see [1], [3], [6] and [9] for information on derivations in Bernstein algebras of order 1).



The set $\mathrm{Der}_K(A)$ of derivations of $A$ is a subspace of the vector space $\mathrm{End}_K(A)$ of the linear transformations of $A$ over $K$. Moreover, $\mathrm{Der}_K(A)$ is a subalgebra of the Lie algebra $\mathrm{End}_K(A)$ and it is called the derivation algebra of the non-associative algebra $A$.

In this section we give necessary and sufficient conditions for a linear transformation of $A$ to be a derivation for both exceptional Bernstein algebras of order 2 and power-associative Bernstein algebras of order 2.

Recall the sets

$$B_{EX} = \{(A,w) | A \ is \ exceptional\},$$

and

$$B_P = \{(A,w) | A \ is \ power-associative\}.$$

**Theorem 1** *Suppose that $A$ is an element of $B_{EX}$. Then, each derivation $D$ on $A$ defines a triple $(\overline{u}, f, g)$, and this triple in fact determines $D$, where $\overline{u} \in U$, $f \in End_K(U)$, $g \in End_k(V_2)$ and for every $u \in U$, $v$, $v' \in V_2$ we have*

*(i)* $4f(e(ex)) + g(x - 4e(ex)) = f(u)v + ug(v) + 4\overline{u}(ex) + 4e(\overline{u}x)$ *for every* $x = uv \in UV_2$,

*(ii)* $4f(e(ey)) + g(y - 4e(ey)) = g(v)v' + vg(v') + 4\overline{u}(ey) + 4e(\overline{u}y) - 4v'[\overline{u}(ev) + e(\overline{u}v)] - 4v[\overline{u}(ev') + e(\overline{u}v')]$ *for every* $y = vv' \in V_2^2$,

*(iii)* $g(ev) = eg(v) + v\overline{u} - 4e(e(v\overline{u}))$ *for every* $v \in V_2$.

*Moreover, $D$ is defined by $D(e) = \overline{u}$, $D(u) = f(u)$ for every $u \in U$ and $D(v) = -4[e(v\overline{u}) + (ev)\overline{u}] + g(v)$ for every $v \in V_2$.*

**Proof:** Let $D$ be a derivation on $A$, then $D(e^2) = 2eD(e)$ and $D(e) \in U$. Let $u \in U$. Then $D(u) = \lambda e + u_1 + v \in A$. Using the definition of $U, U^2 = \{0\}$ and $D \in \mathrm{Der}_K(A)$ we prove that $D(u) = u_1 \in U$. Moreover, since $D$ is a derivation on $A, u_1 = f_D(u)$ where $f_D \in \mathrm{End}_K(U)$. In a similar way, using the fact that for every $v \in V_2$, $e(ev) = 0$, we have

$$e[eD(v)] + e[vD(e)] + [ev]D(e) = 0 \quad (8)$$

Let $D(v) = \beta e + u + v_1$. By using (8) and (4) we can prove that $D(v) = -4[e(vD(e)) + (ev)D(e)] + v_1$. Since $D \in \mathrm{Der}_K(A)$, $v_1 = g_D(v)$ where $g_D \in \mathrm{End}_K(V_2)$. Thus we can prove that $D(v) = -4[e(vD(e)) + (ev)D(e)] + g_D(v)$, for every $v \in V_2$. Using the fact that $D \in \mathrm{Der}_K(A)$, along with the definitions of $D(u)$, and $D(v)$ $\forall u \in U$, $v \in V_2$, relations (2) and (3), we can prove that $4f_D(e(ex)) + g_D(x - 4e(ex)) = f_D(u)v + ug_D(v) + 4\overline{u}(ex) + 4e(\overline{u}x)$ $\forall x = uv \in UV_2$. Since $D \in \mathrm{Der}_K(A)$ the definitions of $D(u)$, and $D(v)$ $\forall u \in U$, $v \in V_2$ and relations (2) and (3) imply that $4f_D(e(ey)) + g_D(y - 4e(ey)) = g_D(v)v' + vg_D(v') + 4\overline{u}(ey) + 4e(\overline{u}y) - 4v'[\overline{u}(ev) + e(\overline{u}v)] - 4v[\overline{u}(ev') + e(\overline{u}v')]$ $\forall y = vv' \in V_2^2$. Finally, using (8) and the fact that $ev \in V$ we can prove that $g_D(ev) = eg_D(v) + vD(e) - 4[e(e(vD(e)))]$ $\forall v \in V_2$.
Therefore, the triple $(D(e), f_D, g_D)$ satisfies the conditions (i), (ii) and (iii) of the Theorem.

Conversely, the linear mapping $D : A \to A$ defined by $D(e) = \overline{u}$, $D(u) = f(u)$ $\forall u \in U$, $D(v) = -4[e(v\overline{u}) + (ev)\overline{u}] + g(v)$ $\forall v \in V_2$ with $f$ and $g$ satisfying the conditions (i), (ii) and (iii) is a derivation on $A$. Using the definition of $D$ together with (i), we can prove that $D(uv) = ug(v) + vf(u)$ for every $u \in U$, $v \in V_2$. Similarly, using (ii) we obtain $D(vv') = vD(v') + v'D(v)$ for every $v, v' \in V_2$. Finally, since $ev \in V_2$ for every $v \in V_2$, relation (iii) implies that $D(ev) = eD(v) + vD(e)$ for every $v \in V_2$.

**Theorem 2** *Suppose that $A$ is an element of $B_P$. Then, each derivation $D$ of $A$ defines triple $(\overline{u}, f, g)$, , and this triple in fact determines $D$, where $\overline{u} \in U$, $f \in End_K(U)$, $g \in End_K(V_2)$ and for every $u$, $u' \in U$, $v$, $v' \in V_2$ we have*

*(a)* $f(uv) = f(u)v + ug(v)$,

*(b)* $g(uu') = uf(u') + u'f(u)$,

*(c)* $g(vv') = vg(v') + v'g(v)$.



*Moreover, $D$ is defined by $D(e) = \overline{u}$, $D(u) = f(u) + 2\overline{u}u$ for every $u \in U$ and $D(v) = -2\overline{u}v + g(v)$ for every $v \in V_2$.*

**Proof:** Let $D$ be a derivation on $A$. Then $D(e^2) = 2eD(e)$ and $D(e) \in U$. Let $u \in U$. Then $D(u) = \lambda e + u_1 + v$ with $\lambda \in K, u_1 \in U$ and $v \in V_2$. Using the fact that $D \in \mathrm{Der}_K(A)$ and the definition of $U$ we can prove that $D(u) = f_D(u) + 2D(e)u$ with $f_D \in \mathrm{End}_K(U)$. In a similar way, using $eV_2 = \{0\}$ and $D \in \mathrm{Der}_K(A)$ we can prove that $D(v) = -2D(e)v + g_D(v)$ for all $v \in V_2$, with $g_D \in \mathrm{End}_K(V_2)$. By using the definition of $D(u)$ and $D(v)$ for every $u \in U$, $v \in V_2$, the fact that $D(e) \in U$ and relation (c) we have that $f_D(uv) = f_D(u)v + uf_D(v)$ for every $u \in U$, $v \in V_2$. Similarly, by using de definition of $D(u)$ for all $u \in U$, $U^2 \subseteq V_2$ and Jacobi's identity in $U$, we can prove that relation (b) holds in $A$. Finally, using $V_2^2 \subseteq V_2$, the definition of $D(v)$ for all $v \in V_2$ and (b) we have relation (c) of the Theorem. So, the triple $(D(e), f_D, g_D)$ satisfies the conditions of the Theorem.

Conversely, let $(\overline{u}, f, g)$ be a triple satisfying relations (a), (b) and (c), then $D(e) = \overline{u}$, $D(u) = f(u) + 2D(e)u$ for every $u \in U$ and $D(v) = -2D(e)v + g(v)$ for every $v \in V_2$ defines a derivation on $A$.

**Corollary 1** *Suppose that $A$ is an element of $B_{EX} \cup B_P$. Then the map $\varphi : Der_K(A) \to U \times End_K(U) \times End_K(V_2)$ defined by $\varphi(D) = (D(e), f_D, g_D)$ for each $D \in Der_K(A)$ is a monomorphism of vector spaces. If $A$ is finite dimensional then dim $(Der_K(A)) \leq r + r^2 + t^2$ where $r = dim(U)$, $t = dim(V_2)$.*

**Corollary 2** *Suppose that $A$ is an element of $B_{EX}$. Then the maps $f : Der_K(A) \to End_K(U)$ defined by $f(D) = f_D$ and $g : Der_K(A) \to End_K(V_2)$ defined by $g(D) = g_D$ are Lie algebra homomorphisms.*

**Corollary 3** *Let $A = Ke \oplus V_2$. Then $Der_K(A) \simeq \{D \in Der_K(V_2) / D(ev) = eD(v),\ \forall v \in V_2\}$. Moreover, if $A$ is an element of $B_P$ then $Der_K(A) \simeq Der_K(V_2)$.*

**Remark 2.** There exist non isomorphic Bernstein algebras of order 2 with isomorphic derivation algebras. In fact, let us consider the following non isomorphic Bernstein algebras of order 2: $A_{17} = Ke\oplus < v_1, v_2, v_3 >$ with multiplication table given by $e^2 = e$, $ev_3 = v_3e = v_1$ $v_1v_3 = v_3v_1 = v_2, v_3^2 = v_1$, all other products being zero and $A_{18} = Ke\oplus < v_1, v_2, v_3 >$ with multiplication given by $e^2 = e$, $ev_3 = v_3e = v_1$, $v_1v_3 = v_3v_1 = v_1$, $v_3^2 = v_2$, all other products being zero (see [8]). Then for every derivation $D$ of $A_i$, $i = 17, 18$ we have $D(e) = 0$, $D(v_1) = D(v_2) = 0$ and $D(v_3) = \alpha v_2$ with $\alpha \in K$. Therefore $Der_K(A) \simeq K^*$.

## 3 Peirce transformations

Let A be an element of $B_P$ and $s$ be an element of $U$. We denote $e_s = e + s + s^2 \in Ip(A)$ and $K_{e_s} \oplus U_s \oplus V_{2s}$ the Peirce decomposition of $A$ relative to $e_s$, where $U_s = \{u_s = u + 2su / u \in U\}$ and $V_{2s} = \{v_s = -2sv + v / v \in V_2\}$ are subspaces of $A$ (see [4] and [7] for details).

For every $s \in U$ let us consider the map $\varphi_s : A \to A$ defined by $\varphi_s = I - 4L_eL_s + 4L_sL_e + 4(L_sL_e)^2$, where $I = id_A$ and $L_x$ is the multiplication by $x$.

For every $s \in U$ the map $\varphi_s$ is an automorphism of vector spaces, called a *Peirce transformation* of $A$. Moreover $\varphi_s(e + u + v) = e_s + u_s + v_s$ for every $u \in U, v \in V_2$.

We remark that the transformation $\varphi_s$ is not an algebra homomorphism, but this is so in the following case:

**Proposition 1** *Suppose that $A$ is an element of $B_P$. Then the following conditions are equivalent:*

*(i) $A \in B_{EX}$.*

*(ii) For every $s \in U$, $\varphi_s - I$ is a derivation on $A$.*

**Proof:** $(i) \Rightarrow (ii)$. Let $D = \varphi_s - I$ with $s \in U$, then $D$ is a linear mapping. Since $A$ is exceptional, using the definition of $\varphi_s$ we have that $D(e) = s$, $D(u) = 0$ and $D(v) = -2sv$ for every $u \in U, v \in V_2$. Moreover, $UV_2 \subseteq U$ and $U^2 = \{0\}$ imply that $D(uv) = uD(v) + vD(u)$ for every $u \in U, v \in V_2$. Since $V^2 \subseteq V_2$, we have that $v_1v_2 \in V_2$ for every $v_1, v_2 \in V_2$. Then,



relation (6) implies that $D(v_1v_2) = -2s(v_1v_2) = -2(sv_1)v_2 - 2(sv_2)v_1 = v_2D(v_1) + v_1D(v_2)$. Finally, it is clear that $D(uu') = uD(u') + u'D(u)$ for every $u, u' \in U$. Therefore $D \in \mathrm{Der}_K(A)$.
$(ii) \Rightarrow (i)$. Suppose $\varphi_s - I \in \mathrm{Der}_K(A)$, for every $s \in U$. Then Theorem 2. implies that $(\varphi_s - I)(e) \in U$. On the other hand, $\varphi_s(e) = e + s + s^2$. Therefore, $(\varphi_s - I)(e) = s + s^2 \in U$. So $s^2 = 0$ for every $s \in U$ and $U^2 = \{0\}$.

**Proposition 2** *Suppose $A \in B_P$, $D$ a derivation on $A$ and $I + D$ a Peirce transformation of $A$. Then $I + D$ is an automorphism of $A$.*

**Proof:** Let $I + D = \varphi_s$ with $s \in U$ then $\varphi_s - I = D$ and by Proposition 1. $\varphi_s$ is an algebra automorphism of $A$.

**Remark 3.** The converse to Proposition 2. is false. For example, let $A$ be the commutative algebra with basis $\{e, u, v_1, v_2\}$ and multiplication table given by $e^2 = e$, $eu = ue = \frac{1}{2}u$, $u^2 = v_1$, $v_2^2 = v_1$, all other products being zero. Then $A$ is a baric algebra satisfying the identity $(x^2)^2 = w(x)x^3$ that charaterizes a power-associative Bernstein algebra of order 2 with $v^3 = 0$ $\forall v \in V_2$, (see [2], Theorem 2.3). Thus, $A \in B_P$. If we define $d : A \to A$ by $d(v_2) = v_1$ and all other products being zero, then $d$ is a derivation on $A$. Moreover $\sigma = I + d$ is an automorphism of $A$, but $\sigma \neq \varphi_x$ for all $x \in A$, because $\sigma(e) = e$ and $\varphi_{\lambda u}(e) = e + \lambda u + \lambda^2 v_1$, for every $\lambda \in K$.

**Corollary 4** *Let $A \in B_{EX} \cap B_P$, then $\{ \varphi_s - I \ / \ s \in U \}$ is the trivial Lie subalgebra of $Der_K(A)$.*

**Proof:** Since $U^2 = \{0\}$ we have that $\varphi_s \circ \varphi_{s'} = \varphi_{s+s'}$, for every $s, s' \in U$ Therefore $[\varphi_{s'} - I, \varphi_s - I] = 0$, for every $s$, $s' \in U$, as desired.

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# EXTENSIONS PONDÉRÉES D'ALGÈBRES

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**Abstract**

Nous construisons des plongements "canoniques", par adjonction d'un élément, d'une algèbre quelconque dans des algèbres pondérées. Nous donnons une description des extensions possibles.

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