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Orthogonality in WBJ algebras. (English summary)
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Let A be a finite-dimensional commutative algebra over an infinite field K, char $K \neq 2$. If $\omega\colon A \to K$ is a nonzero algebra morphism, the pair (A,ω) is called a baric algebra. A is called a weak Bernstein-Jordan (WBJ) algebra if it has a decomposition $A = Ke \oplus S$, where e is an idempotent and for all $x \in S$, $ex = 2e(ex)$, $x^2 = ex^2 + 2(ex)x$ and $x^3 = 0$. If $A = Ke \oplus U \oplus V$ is the Peirce decomposition of a WBJ-algebra with respect to an idempotent e, then $U^2 \subset V$, $UV \subset U$, $V^2 \subset Ke$ and the following relations are satisfied: For all $u, u' \in U$, $v \in V$, $v' \in V_\perp = \mathrm{Ann}(V)$ and $v^2 = \lambda e$, with $\lambda \in K$, $(uu')v = 0$, $u(u'v) + u'(uv) = 0$, $v(uv') + v'(uv) = 0$, $v'(uv') = 0$, $v(uv) = -\frac{1}{4}\lambda u$, $(uv')^2 = 0$, $(uv)^2 = \frac{1}{4}\lambda u^2$, $(uv)(uv') = 0$. If $V^2 = 0$, A is a Bernstein-Jordan algebra. A is said to be orthogonal if $U^3 = 0$, to be quasiorthogonal if $U^2(UV) = 0$, and to be strongly orthogonal if $U^3 = 0$ and $U(UV) = 0$.

In this paper the authors study orthogonality, quasiorthogonality and strong orthogonality in WBJ-algebras. They show that every WBJ-algebra of dimension strictly less than 9, 6 or 5 is respectively quasiorthogonal, orthogonal or strongly orthogonal. Because in the case $V^2 = 0$ the concepts of orthogonality and quasiorthogonality coincide (A is then a Bernstein-Jordan algebra), every WBJ-algebra satisfying $V^2 = 0$ of dimension strictly less than 11 or 7 is respectively quasiorthogonal or strongly orthogonal. In each case the authors show that the dimension established is the best possible.

No characterization of quasiorthogonal, orthogonal or strongly orthogonal WBJ-algebras is given. *Moussa Ouattara*