SYMMETRY AND THE MONSTER
One of the Greatest Quests of Mathematics

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Chapter 2. Galois: Death of a Genius

In Paris on the evening of 29 May 1832 the young French mathematician Évariste Galois wrote a letter he knew would be the last of his life

Though his fame as a revolutionary was transient, his mathematics was timeless: Galois groups are common currency in mathematics today.

As a young man of 20, he joined the ranks of the immortals. How is this possible?

When Galois was held back by the headmaster against his father's will, the effect was devastating and the 15 year old started rejecting everything but mathematics.

The conflict between Galois' father and the headmaster was part of a wider political problem.

Galois' main ideas concerned the solution of algebraic equations

The quadratic formula is ancient, first discovered by the Babylonians in about 1800 BCE, nearly 4000 years ago (They wrote in words rather than symbols)

A general method for dealing with equations of degree 3 had to wait nearly 3000 years until Omar Khayyám (1048–1134), the famous Persian mathematician and astronomer (better known for his poetry), devised a geometric method

A numerical formula was found 400 years later during the Italian Renaissance for equations of degree 3 and 4. In the early 16th century four Italian mathematicians moved algebra into a new era

In retrospect, these four were men of genius and "constituted the most singular team in the whole history of science."

However, no one could find a recipe for solving equations of degree 5

In 1799, no less an authority than Gauss wrote "Since the works of many geometers left very little hope of ever arriving at the resolution of the general equation algebraically, it appears increasingly likely that this resolution is impossible and contradictory"

That same year, this "conjecture" was "confirmed" and published in a 500 page book. However, this work was never fully accepted

The matter was finally settled in 1824 when a young Norwegian mathematician, Niels Hendrik Abel (1802–1829) produced an independent proof

Abel showed that there were some equations of degree 5 whose solutions could not be extracted using square roots, cube roots, fourth and fifth roots, etc

But the problem was to decide which equations could be solved in this way, and which couldn't. This set the stage for the entrance of Évariste Galois, who died even younger than Abel.

Galois measured the amount of symmetry between the various solutions to a given equation and used it in an imaginative new way

Galois' ideas for using symmetry were profound and far-reaching, but none of this was fully understood at the time, and political events were overtaking his work

Rejected by the academic establishment, rejected by the state, rejected in romance, and losing the father he loved, there remained only the republican ideals to satisfy his anger

Galois: if a body was needed, it should be his. He would arrange a duel and a riot would take place at his funeral. (The dual took place, but not the riot at his funeral)

Galois' death at 20 achieved nothing for the revolution. For mathematics, however, his achievements will last forever