$$(\alpha,x)\big((\alpha,x)^2(\beta,y)\big) = (\alpha,x)\big((\alpha^2+q(x),\, 2\alpha x)(\beta,y)\big)$$

$$= (\alpha,x)\big((\alpha^2+q(x))\beta + q(2\alpha x, y),\ (\alpha^2+q(x))y + 2\beta\alpha x\big)$$

$$= \Big(\alpha(\alpha^2+q(x))\beta + \alpha q(2\alpha x, y) + q\big(x, (\alpha^2+q(x))y + 2\beta\alpha x\big),$$

$$\alpha(\alpha^2+q(x))y + 2\alpha^2\beta x + \big((\alpha^2+q(x))\beta + q(2\alpha x, y)\big)x\Big)$$

$$(\alpha,x)^2\big((\alpha,x)(\beta,y)\big) = (\alpha^2+q(x),\, 2\alpha x)(\alpha\beta + q(x,y),\, \alpha y + \beta x)$$

$$= \Big((\alpha^2+q(x))(\alpha\beta + q(x,y)) + q(2\alpha x, \alpha y + \beta x),\ (\alpha\beta + q(x,y))2\alpha x$$

$$+ (\alpha^2+q(x))(\alpha y + \beta x)\Big)$$

Thus $(\alpha,x)\big((\alpha,x)^2(\beta,y)\big) = (\alpha,x)^2\big((\alpha,x)(\beta,y)\big)$

so D is a Jordan algebra.

Shorter proof: $\cancel{L(z^2)(\beta,y) = z^2(\beta,y) = (\alpha^2+q(x), 2\alpha x)(\beta,y)}$

~~if $z=(\alpha,x)$~~ See the next page

7

$$\left(+\alpha^2 + q(x), 0\right) + 2\alpha(\alpha, x)$$

$$= \left(+\alpha^2 + q(x), 0\right) + \left(2\alpha^2, 2\alpha x\right)$$

$$= \left(3\alpha^2 + q(x), 2\alpha x\right) =$$

$$= \left(\alpha^2 + q(x), 2\alpha x\right) + \left(2\alpha^2\right.$$

$$2\alpha z + \left(\alpha^2 + q(x), 0\right)$$

$$= 2\alpha(\alpha, x) + (\qquad)$$

$$= \left(2\alpha^2, 2\alpha x\right) + \left(\alpha^2 + q(x), 0\right)$$

$$= \left(3\alpha^2 + q(x), 2\alpha x\right)$$

$$= \left(\alpha^2 + q(x), 2\alpha x\right) + \left(2\alpha^2, 0\right)$$

$$= z^2 + 2\alpha^2 \cdot (1, 0)$$

$$\therefore \quad z^2 = 2\alpha z + \left(-\alpha^2 + q(x)\right) 1$$

$$L(z)L(z^2) = 2\alpha L(z)^2 + \left(q(x) - \alpha^2\right) L(z)$$

$$\boxed{\because L(z)L(z^2) = L(z^2)L(z)}$$

$$L(z^2)L(z) = 2\alpha L(z)^2 + \left(q(x) - \alpha^2\right) L(z)$$

$$\boxed{2\alpha z + \left(-\alpha^2 + q(x)\right) 1 = 2\alpha(\alpha, x) + \left(\left(-\alpha^2 + q(x)\right)1, 0\right) = \left(2\alpha^2, 2\alpha x\right) + \left(-\alpha^2 + q(x), 0\right) = \left(\alpha^2 + q(x), 2\alpha x\right) = z^2}$$

$$\boxed{L(z^2) = 2\alpha L(z) + \left(q(x) - \alpha^2\right) L(1)}$$

chpt 7 One proof is 1 & 2 1-15-17

Exercise 2 p. 67 a second proof is 3

let

$$\left[[L(x), L(y)], L(u) \right] = L(x(yu) - y(xu)) \qquad (7.4)$$

let $D = [L(x), L(y)]$ 1

$DL(u) - L(u)D =$

$L(L(x)(yu))$

$- L(L(y)(xu))$

$= L(L(x)L(y)u - L(y)L(x)u)$

$= L(Du)$

~~$D(ab) = D(L(a)b)$~~

~~$[D, L(z)] =$~~ OR 3 alternate proof

$$D(zw) \overset{?}{=} z D(w) + (Dz) w$$

$$\| \qquad\qquad \|$$

$$DL(z)(w) \overset{?}{=} L(z)D(w) + D(z) \cdot w$$

$$[D, L(z)](w) \overset{?}{=} L(D(z))(w)$$

$$\|$$

$$\left[[L(x), L(y)], L(z) \right] \overset{?}{=} L(D(z))$$

$$\|$$

$$L(x(yz)) - L(y(xz))$$

$$\|$$

$$L(L(x)L(y)z - L(y)L(x)z)$$

$$\|$$

$$L(Dz)$$

QED

2

$D(ab) =$

$D(L(a)b)$

$= L(a)Db$

$+ (Da)b$

$= aDb + (Da)b$

Discussion of Exercise 3 chapter 7

(7.7) $L(y)P(x) + P(x)L(y) = P(xy, x)$

From the definition of $P(x)$,
~~From p. (3) of 1/24/17~~ (7.7) is the same as

$$\boxed{\begin{aligned}&2L(y)L(x)^2 - L(y)L(x^2) + 2L(x)^2L(y) - L(x^2)L(y)\\ &= 2L(x)L(xy) + 2L(xy)L(x) - L(2x(xy))\end{aligned}}$$ (*)

replace x by $u+w$

$$2L(y)\left(L(u)^2 + L(u)L(w) + L(w)L(u) + L(w)^2\right)$$
$$- L(y)\left(L(u^2) + L(2uw) + L(w^2)\right)$$
$$+ 2\left(L(u)^2 + L(u)L(w) + L(w)L(u) + L(w)^2\right)L(y)$$
$$- \left(L(u^2) + L(2uw) + L(w^2)\right)L(y)$$

$$= 2\left(L(u)+L(w)\right)\left(L(uy)+L(wy)\right) + 2\left(L(uy)+L(wy)\right)\left(L(u)+L(w)\right)$$
$$-2L\left(\cancel{\ }(u+w)(uy+wy)\right) =$$
$$"\ -2\left(L(u(uy)) + L(w(uy)) + L(u(wy)) + L(w(wy))\right)$$

$$2L(y)L(u)^2 + 2L(y)L(u)L(w) + 2L(y)L(w)L(u) + 2L(y)L(w)^2$$
$$- L(y)L(u^2) - 2L(y)L(uw) - L(y)L(w^2)$$
$$+ 2L(u)^2L(y) + 2L(u)L(w)L(y) + 2L(w)L(u)L(y) + 2L(w)^2L(y)$$
$$- L(u^2)L(y) - 2L(uw)L(y) - L(w^2)L(y)$$

$$= 2L(u)L(uy) + 2L(w)L(uy) + 2L(u)L(wy) + 2L(w)L(wy)$$
$$+ 2L(uy)L(u) + 2L(wy)L(u) + 2L(wy)L(w) + 2L(uy)L(w)$$
$$- 2L(u(uy)) - 2L(w(uy)) - 2L(u(wy)) - 2L(w(wy))$$

The terms lableled 1 cancel by $\ast$
The terms lablel 2 cancel by $\ast$

What remains after cancelling the 2 is

$$L(y)L(u)L(w) + L(y)L(w)L(u) - L(y)L(uw)$$
$$+ L(u)L(w)L(y) + L(w)L(u)L(y) - L(uw)L(y)$$

$$= L(w)L(uy) + L(u)L(wy) + L(wy)L(u)$$
$$+ L(uy)L(w) - L(w(uy)) - L(u(wy))$$

apply to v

$$\boxed{\begin{aligned} & y(u(wv)) + y(w(uv)) - y((uw)v) \\ & + u(w(yv)) + w(u(yv)) - (uw)(yv) \\ = {} & w((uy)v) + u((wy)v) + (wy)(uv) \\ & + (uy)(wv) - (w(uy))v - (u(wy))v \end{aligned}}$$

let's rewrite (7.10) using $\{abc\} = P(a,c)\,b$

where by (7.6) $P(x,y) = 2\big(L(x)L(y) + L(y)L(x) - L(xy)\big)$

(7.10) $y\{uvw\} \overset{?}{=} \{(yu)vw\} - \{u(yv)w\} + \{uv(yw)\}$

(7.10') $\underbrace{y(P(u,w)v)}_{LHS} \overset{?}{=} \underbrace{P(yu,w)v}_{RHS_1} - \underbrace{P(u,w)(yv)}_{RHS_2} + \underbrace{P(u,yw)v}_{RHS_3}$

~~(7.10'')~~ To prove (7.10'), we can forget about the 2 in 7.6 !!

$$LHS = \cancel{2}\,y\big((L(u)L(w) + L(w)L(u) - L(wu))v\big)$$

$$= \boxed{\cancel{2}\,y(u(wv)) + y(w(uv)) - y((wu)v)}$$

voila!

$RHS_1 = \cancel{=}$ etc !!!

Exercise 4 chpt 7

Exercise 4 ch 7. pdf

2/24/17

$$D\{uvw\}^{*} = D\left((uv)w + (wv)u - (uw)v\right)$$

$$\{Du, v, w\} = ((Du)v)w + (wv)Du - ((Du)w)v$$

$$\{u\, Dv\, w\} = (u\,Dv)w + (w(Dv))u - (uw)(Dv)$$

$$\{u\, v\, Dw\} = (uv)\,Dw + ((Dw)v)u - (u(Dw))v$$

$$\left\{\begin{array}{l} D\,L(a) - L(a)D = L(Da) \\ D(ab) = a\,Db + (Da)b \end{array}\right\} \text{ def. of derivation}$$

$$D((uv)w) = (uv)\,Dw + D(uv)\,w$$
$$= (uv)\,Dw + (u\,Dv)w + ((Du)v)w$$

$$D((wv)u) = (wv)\,Du + D(wv)u$$
$$= (wv)\,Du + (w\,Dv)u + ((Dw)v)u$$

$$D(uw)v = (uw)\,Dv + D(uw)v$$
$$= (uw)\,Dv + (u\,Dw)v + ((Du)w)v$$

$$^{*}\{uvw\} = P(u,w)v \overset{(7.6)}{=} 2\left(L(u)L(w) + L(w)L(u) - L(uw)\right)v$$

$$= 2\,u(wv) + 2\,w(uv) - 2(uw)v$$

ch7ex5NOTES.pdf 2/1/17 (1)

Exercise 5 chpt 7

$$(7.11)\quad \{xy\{uvw\}\} - \{uv\{xyw\}\} = \{\{xyu\}vw\} - \{u\{yxv\}w\}$$

$x,y \to \{uvu\} = 2P(u)v$, $w \to u$

recall $P(x,z)y = \{xyz\}$
$P(x,x) = 2P(x)$

$$\{2P(u)v, y, 2P(u)v\} - \{uv\{2P(u)v, y, u\}\} = \{\{2P(u)v, y, u\}vu\}$$

these are equal

$= 4 \cdot 2\, P(P(u)v)y$

$$- \{u\{y, 2P(u)v, v\}u\}$$

$$= -\ 2P(u)\{y, 2P(u)v, v\}$$

$$(7.15)\quad 8\, P(P(u)v)y = 2\{uv\{uy\{uvu\}\}\} - \{u\underbrace{\{y\{uvu\}v\}}u\}$$

$$(7.13)\quad \{\{xyu\}vw\} - \{u\{yxv\}w\} = \{x\{vuy\}w\} - \{\{uvx\}yw\}$$

$x \to v,\ y \to u,\ u \to y,\ v \to u,\ w \to v$

$$\underbrace{\{\{vuy\}uv\}} - \{y\{uvu\}v\} = \{v\{uyu\}v\} - \underbrace{\{\{yuv\}uv\}}$$

equal

$$(7.13')\quad \boxed{\{y\{uvu\}v\}} = 2\{\{vuy\}uv\} - \{v\{uyu\}v\}$$

$$(7.13)'' \quad \{u\{y\{uvu\}v\}u\} = 2\{u\{\{vuy\}uv\}u\} - \underbrace{\{u\{v\{uyu\}v\}u\}}_{=8P(u)P(v)P(u)y}$$

substitute (7.13)'' into (7.15)

$$8P(P(u)v)y = 2\{uv\{uy\{uvu\}\}\} - \boxed{(2\{u\{\{vuy\}uv\}u\} - 8P(u)P(v)P(u)y)}$$

$$(7.9) \quad L(x,y)P(x) = P(x)L(y,x) = P(P(x)y, x)$$

$$\tfrac{1}{2}\{xy\{xzx\}\} = \tfrac{1}{2}\{x\{yxz\}x\} = \{P(x)y, z, x\}$$

$$\text{''}\quad \tfrac{1}{2}\{\{xyx\}, z, x\}$$

u u v u

$$\{uy\{uvu\}\} = \{\{uyu\}v, u\} = \{u\{yuv\}u\}$$

$$\{uv\{uy\{uvu\}\}\} = \{uv\{u\underbrace{\{yuv\}}_{Y}u\}\} \overset{?}{=} \{u\{\{vuy\}uv\}u\}$$

Need to prove

then you get $P(P(u)v) = P(u)P(v)P(u)$ (7.14)

$$\{uv\{uYu\}\} = \{u\{Yuv\}u\}$$

$$\therefore \{uv\{u\{yuv\}u\}\} = \{u\{\{yuv\}uv\}u\} \qquad \text{IT WORKS!}$$