MATH 3A MIDTERM–**A**

FEBRUARY 6, 2015

10:00AM–11:50AM

Instructor: Guangbo Xu

"I pledge on my honor that I haven't received or given any unauthorized assistance on this exam."

Print Your Name :

Signature :

Problem	Points
I	
II	
III	
Total	

Requirements & Suggestions:

- This exam is close-book. You are only allowed to use pens, pencils, erasers, and calculators without scientific functions.
- For each problem, make sure you understand the statement. Inform the instructor to clarify anything you are not sure of.
- Write your solutions with clear structure. Show all your work. It's better to do the problems first on the scratch paper.
- Do not turn in the exam during the last 10 minutes of the exam.
- When turning your exam, we will check your ID.

I.(24 points) True or False: For each statement, determine whether it is true or false (2 points), and explain your answer (2 points).

(1) The matrix $\begin{bmatrix} 1 & 0 & 3 & 0 & 4 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ is in reduced row echelon form (RREF).

(2) Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be vectors in $\mathbb{R}^n$. If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly independent, then $\mathbf{w}$ is not equal to any multiple of $\mathbf{v}$.

(3) If A, B are $n \times n$ matrices, then $(A+B)(A-B) = A^2 - B^2$.

(4) If in a linear system, the number of variables is bigger than the number of equations, then it has infinitely many solutions.

(5) If a linear system has at least two different solutions, then it must has infinitely many solutions.

(6) If a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ is onto, then it is also one-to-one.

II.(24 points) Consider the following system of linear equations

$$\begin{cases} x_1 & & +x_3 & = 2 \\ 3x_1 & +x_2 & +4x_3 & = 4 \\ 3x_1 & -3x_2 & +1x_3 & = 0 \end{cases}$$

Answer the following questions. Make your solution as complete as possible.

(1) Write down the augmented matrix of this system and use row reduction algorithm to transform it to the reduced row echelon form.

(This problem is continued on next page.)

(2) Is this system consistent? If yes, write down the general solution.

(3) Let A be the coefficient matrix of the system. Find its inverse, and use the inverse to solve the following equation.

$$A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -4 \end{bmatrix}$$

III.(12 points) Consider the vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} h \\ 3-h \\ -1 \end{bmatrix}.$$

For what value(s) of h is $\mathbf{y}$ in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$? For each of those specific values of h, write $\mathbf{y}$ as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.