HOMEWORK 1

Due Thursday, April 13, at the beginning of discussion

1. For a finite set S of size $|S| = s$, prove that the size of the power set of S is $|\mathcal{P}(S)| = 2^s$.
2. Prove that a function $f : A \to B$ is injective if and only if f has a left-inverse. (f has a left-inverse means: there is a function $g : B \to A$ such that $g \circ f = \mathrm{id}_A$). Give an example of a function that has a right-inverse but no left-inverse.
3. Write down all (complex) solutions to the equation $z^8 = 1$.
4. (a) Write down a 2×2 matrix T which flips a vector $\vec{v}$ around the y-axis.
   (b) Write down a 2×2 matrix R_θ which will rotate a vector $\vec{v}$ counter-clockwise by an angle θ. i.e. $R_\theta \vec{v}$ will be the rotated version of $\vec{v}$. For example, if $\theta = \pi/4$, then $R_\theta \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right) = \frac{1}{\sqrt{2}} \left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right)$. Explain how you found the matrix. (the entries of R_θ should depend on θ).
5. Show that a finite union of countable sets is countable.
6. Show that every map $f : A \to B$ of sets decomposes as $h \circ g$ where $g : A \to C$ is surjective and $h : C \to B$ is injective.
7. Recall that, for $n \in \mathbb{Z}$, $n > 0$, there is a relation on Z called congruence mod n. The definition is that $a \sim b$ if and only if n divides $a - b$ (i.e. there exists a $c \in \mathbb{Z}$ such that $cn = a - b$). Prove that $\sim$ is an equivalence relation.