HOMEWORK 2

Due Thursday, April 20, at the beginning of discussion

1. Let $n \in \mathbb{N}$. Find all complex solutions to the equation: $z^n = i$.
2. In this problem, you will prove the sin and cos sum formulas in two ways.

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

   - Use Euler's formula: $e^{ia} = \cos a + i \sin a$ to prove the formulas.
   - Use the rotation matrix R_θ from last term and the fact that the matrix of a composition of two linear transformations is the product of their respective matrices. (therefore $R_\theta R_\nu = R_{\theta+\nu}$)
3. Prove Euler's formula $e^{ia} = \cos a + i \sin a$ by using the power series expansion of e^x, $\sin x$ and $\cos x$.
4. Fill the table for the binary operation in a way that makes the operation associative.

*	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	d	c	d
d				

5. Let $M_2(\mathbb{R})$ be the set of 2×2 matrices with real entries, and let K be the subset of $M_2(\mathbb{R})$ defined by

$$K = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a,\, b \in \mathbb{R} \right\}.$$

   - Show that addition of matrices is a binary operation on K.
   - Is $(K, +)$ a group? Prove your answer.
   - Show that $(K, +)$ is isomorphic to $(\mathbb{C}, +)$. (You need to build a map that associates a complex number to each matrix in K, and you mush show that your map is an isomorphism.)
   - Show that multiplication of matrices is a binary operation on K.
   - Is $(K, *)$ a group? Prove your answer.
   - Show that $(K, \cdot)$ is isomorphic to $(\mathbb{C}, \cdot)$.

6. Let U be a set and let X be the power set of U (that is, the set of all subsets of U). Consider the operation of *symmetric difference* of sets, defined by

$$A \triangle B = (A \cup B) - (A \cap B) = (A - B) \cup (B - A).$$

The operation of symmetric difference is a binary operation on X.

a) Show that $\triangle$ is commutative.

b) Is there an identity element?

c) Does every set A have an inverse? What is it?

d) Is $(U, \triangle)$ a group?