# HOMEWORK 7

*Due Thursday, June 8, at the beginning of discussion*

1. Is $S_3 \times S_3$ isomorphic to either $D_{12}$ or $D_{36}$? Prove your answers.
2. Make a list of all homomorphisms $\mathbb{Z}/8\mathbb{Z} \to \mathbb{Z}/8\mathbb{Z}$. Prove that this is a complete list.
3. (a) Assume $\varphi : G \to H$ is a homomorphism and $G$ is a finite group. If the image of $\varphi$ has an element of order $n$, prove that $G$ has an element of order $n$.
   (b) Give an example to show that this result does not hold if $G$ is allowed to be an infinite group.
4. Prove or disprove: If $d$ divides $|G|$, then $G$ has an element of order $d$.
5. Denote the center of $D_4$ by $Z(D_4)$.
   (a) Explicitly list the elements of $D_4/Z(D_4)$.
   (b) Determine the order of the elements of $D_4/Z(D_4)$.
   (c) Is $D_4/Z(D_4)$ isomorphic to $\mathbb{Z}_4$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$?
6. Consider that the map
$$\phi : \mathbb{Z}_8 \to \mathbb{Z}_2,\ \bar{k} \mapsto \bar{k}.$$
   (a) Compute the Kernel of $\phi$ and the Image of $\phi$.
   (b) Find a familiar group isomorphic to $\mathbb{Z}_8/Ker(\phi)$. Hint: Use the first isomorphism theorem (also known on the south side as fundamental homomorphism theorem).
7. Let $G = \mathbb{Z}_4 \times \mathbb{Z}_6$ and $H = \langle \bar{2} \rangle \times \langle \bar{4} \rangle$.
   (a) Explicitly list the elements of $G/H$.
   (b) Determine the orders of the elements of $G/H$.
   (c) Find a familiar group isomorphic to $G/H$.
8. Match the following quotient groups
   - $\mathbb{R}/\mathbb{Z}$
   - $\mathbb{R}/\mathbb{Q}$
   - $\mathbb{Q}/\mathbb{Z}$

   to their corresponding properties
   - Every element in this group has finite order.
   - This group has non-identity elements of finite order and elements of infinite order.
   - Every non-identity element in this group has infinite order.
9. Prove that $SL_n(\mathbb{R})$ is a normal subgroup in $GL_n(\mathbb{R})$, and that the quotient group $\frac{GL_n(\mathbb{R})}{SL_n(\mathbb{R})}$ is isomorphic to $\mathbb{R}^\times$.
10. Prove that $\mathbb{Z} \times \mathbb{Z}/\langle(2,2)\rangle$ is an infinite group but is not an infinite cyclic group.