Lecture 2: The basics.

- We will use the formalism of <u>sets</u> (like most of mathematics)

  A set S is made up of elements.

  $$S = \{ 1, 2, \underbrace{\{1,2\}}_{\text{this set is an element of } S}, \bigstar, \square \}$$

  or $S = \emptyset$ empty set.

  <u>Ex:</u> $\mathbb{Z}$, $\mathbb{N}$, $\mathbb{R}$ are sets.

- <u>Subsets</u>: $A \subset B$ ✓

  ⚠ "$\subset$" and "$\subseteq$" mean the same thing.

  means
  $\forall a \in A, a \in B.$
  or:
  $\forall a \cancel{\in A}, a \in A \Rightarrow a \in B.$

  There's also: $\subsetneq$ subset but not equal.

- <u>Ex:</u> $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

- Set-building notation:

  $$S = \{ x \in \mathbb{Z} \mid x \text{ is prime} \}$$

  $$S = \{ x \in \mathbb{Q} \mid x^2 < 1 \}.$$

  all the x's in $\mathbb{Q}$ — such that — condition holds.

  $$S = \{ 2n \mid n \in \mathbb{Z} \}.$$

  things of the form $2n$ — for $n \in \mathbb{Z}$.

- number of subsets of S

  is $2^{|S|}$ ← size of S.

- <u>cartesian product</u>:

  $$A \times B = \{(a,b) \mid a \in A,\ b \in B\}.$$

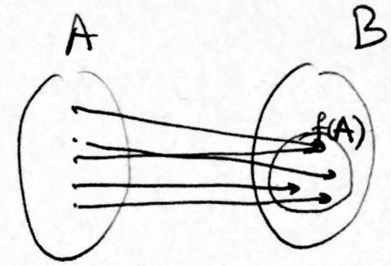
  has $|A||B|$ elements

  $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$ is the plane.

- <u>Functions</u>: $f : A \to B$ is an assignment of an element $f(a) \in B$ to each element $a \in A$.

  domain ↙ ↙ co-domain

  $$f : A \longrightarrow B$$



$$f(A) = \{f(a) \mid a \in A\}$$
$$= \{b \in B \mid \exists a \in A,\ b = f(a)\}$$

is called the <u>image of f</u> or <u>range of f</u>.

Everything is a set, how are functions sets?

$f_{:A \to B}$ is secretly a subset of $A \times B$.

it's the subset

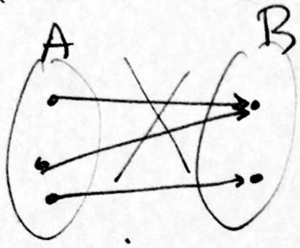
$$\{ (a, f(a)) \mid a \in A \} \subset A \times B.$$

$f: A \to B$

- f is <u>injective</u> or <u>1-1</u> iff (or a <u>monomorphism</u>)

if $\forall a_1, a_2 \in A$,

$$f(a_1) = f(a_2) \Rightarrow a_1 = a_2.$$



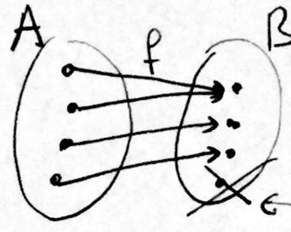
"no two go to the same one".

$:A \to B$

- f is <u>surjective</u> or <u>onto</u> or an <u>epimorphism</u>

if $\forall b \in B$, $\exists a \in A$ s.t. $f(a) = b$.

(equivalently: $f(A) = B$).



"no $b \in B$ misses out".

- f is <u>bijective</u> or a <u>one-to-one correspondence</u> or is an isomorphism (of sets) if f is one-to-one and onto: nicely matched.