

# SOUTHERN CALIFORNIA NUMBER THEORY DAY, NOVEMBER 13, 2010

## ABSTRACTS

Alina Bucur, *Multiple Dirichlet series*

Samit Dasgupta, *On Greenberg's conjecture on derivatives of  $p$ -adic  $L$ -functions with trivial zeroes*

In 1991, Ralph Greenberg stated a conjecture about  $p$ -adic  $L$ -functions that have a trivial zero at  $s = 1$ . Here “trivial” means that the zero arises from the vanishing of an Euler factor that must be removed in order to state the interpolation property of the  $p$ -adic  $L$ -function. Greenberg's conjecture concerns the value of the derivative of the  $p$ -adic  $L$ -function at  $s = 1$ . An example of this conjecture is the case of the  $p$ -adic  $L$ -function of an elliptic curve  $E/\mathbf{Q}$  with split multiplicative reduction at  $p$ . In this case, Greenberg's conjecture reduces to an earlier conjecture by Mazur, Tate, and Teitelbaum, and was proven by Greenberg himself in joint work with Glenn Stevens. In this talk, we will describe a strategy to prove new cases of Greenberg's conjecture. We will concentrate on the case of the symmetric square of an elliptic curve with good reduction at  $p$ . The strategy is a generalization of my previous work with Darmon and Pollack proving certain cases of the Gross-Stark conjecture (which can also be viewed as a special case of Greenberg's conjecture). The method involves studying explicit  $p$ -adic families of modular forms on  $\mathrm{GSp}_4$  and their associated Galois representations.

Kiran Kedlaya, *Relative  $p$ -adic Hodge theory*

In  $p$ -adic Hodge theory, one tries to capture the interrelationships between different cohomology spaces associated to varieties over  $p$ -adic fields (notable de Rham and étale cohomology) through  $p$ -adic analytic constructions. One part of this subject is an analytic description of the category of continuous representations (on finite-dimensional  $p$ -adic vector spaces) of the absolute Galois group of a  $p$ -adic field, through Fontaine's theory of  $(\phi, \Gamma)$ -modules. We outline a similar theory that can be used to describe the fundamental group of a fairly general analytic space over a  $p$ -adic field. Joint work with Ruochuan Liu.

Kristin Lauter, *A Gross-Zagier formula for quaternion algebras over totally real fields*

The values of the elliptic modular  $j$ -function at imaginary quadratic numbers are called singular moduli. They generate the Hilbert class field of the imaginary quadratic field and are of fundamental importance in the study of elliptic curves and in algebraic number theory, including the study of elliptic curves over finite fields. The formula of Gross and Zagier for the factorization of the norm of differences of singular moduli can be viewed as a solution to the problem of counting simultaneous embeddings of the rings of integers of two imaginary quadratic fields into a maximal order in the quaternion algebra ramified only at  $p$  and infinity. In this talk I will describe results generalizing Gross and Zagier's formula to counting simultaneous embeddings of the rings of integers of two primitive quartic CM fields into certain orders in a quaternion algebra over a totally real field. This result has applications to the problem of constructing genus 2 curves for use in cryptography. Joint work with Eyal Goren.