MIDTERM: GEOMETRY OF LINES AND PLANES

Fried, Mon. Oct. 30, 1995

1. Comparing two lines. 45: Consider two lines

$L_1 = \{(2,1,3)+t(v_x,v_y,v_z) \mid t \in R\}$ and $L_2 = \{(x_0,y_0,z_0)+s(2,1,0) \mid s \in R\}$.

In this problem, you choose values of (v_x,v_y,v_z) and (x_0,y_0,z_0) to construct lines with certain properties.

7 a) For what vectors (v_x,v_y,v_z) is L_1 parallel to L_2?

13 b) Suppose $(x_0,y_0,z_0) = (2,1,3)$. Let V be the set of all nonzero vectors (v_x,v_y,v_z) for which L_1 is perpendicular to L_2? Find two vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ in V so $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are perpendicular to each other.

10 c) Suppose $(v_x,v_y,v_z) = (1,3,2)$, and L_1 and L_2 meet at a point. Find a vector $\boldsymbol{w}$ and a point (x_1,y_1,z_1) so the plane

$$P = \{(x,y,z) \mid \boldsymbol{w} \cdot ((x,y,z) - (x_1,y_1,z_1)) = 0\}$$

contains all points of both L_1 and L_2.

15 d) Suppose $(v_x,v_y,v_z) = (1,3,2)$. For what points (x_0,y_0,z_0) is there a plane P that contains both L_1 and L_2?

2. Intersecting planes. 30: Consider $P = \{(x,y,z) \mid 2x - 3y - z = 4\}$.

5 a) Find a vector $\boldsymbol{w}$ and a point (x_1,y_1,z_1) so that P is the set of points $\{(x,y,z) \mid \boldsymbol{w} \cdot ((x,y,z) - (x_1,y_1,z_1)) = 0\}$.

5 b) What vector is the projection of $\boldsymbol{w}$ on the (y,z)-plane? Note: The (y,z)-plane is the set of points where $x = 0$.

10 c) Find (x_0,y_0,z_0) and $\boldsymbol{v} = (v_x,v_y,v_z)$ so $L = \{(x_0,y_0,z_0) + t\boldsymbol{v} \mid t \in R\}$ is the parametric line of intersection of P and the (y,z)-plane.

10 d) The line L is sitting in the (y,z)-plane. The (y,z)-plane is like the blackboard for computing slopes in High School; except you have y replacing x and z replacing y. What is the slope of L in this plane?

3. Planes meeting the graph of a function. 25: Consider the function $f(x,y) = 2x^2 - 3y^2$. Let P be the plane parallel to the (x,z)-plane going through the point $(1,1,-1)$.

10 a) Describe the graph of f.

10 b) Describe the intersection C of P and the graph of f. Further, find $g : R^1 \to R^3$ by $t \mapsto (g_1(t), g_2(t), g_3(t))$ so the range of g is C.

5 c) Find a vector $\boldsymbol{w}$ so the line $L = \{(1,1,-1) + t\boldsymbol{w} \mid t \in R\}$ is tangent to the curve C at the point $(1,1,-1)$.