PROBLEM PRESENTATION

Recurring Curriculum Concepts

Certain themes enter Mathematics in elementary school, and cause trouble throughout the curriculum—even through the college years. Here are some examples of these *recurring curricular concepts*.

(1) How to distinguish between the perimeter and the area of a plane figure; is there area without perimeter?

(2) How to separate the concept of function from analytic rule; do we have to know the rule?

(3) How to represent infinity: where is the end of the number line?; what is the "last" decimal digit; what is the meaning of knowing anything when there is an infinite amount we don't know?

(4) How to construct understanding through mathematical inference; is it necessary?

(5) How to pick an instance, example, specimen at *random*; is it possible? What is a good substitute for it?

(6) How to model giving directions; what are the efficient ways of presenting *direction* and *paths* to understanding? (Vector presentation of a line in 3-space is a good model for this. This is not one tiny topic.)

World Wallpaper: Johnny B. Badde, rock entertainer and world traveler wanted to cover one huge wall of his room with unusual wallpaper. He wanted wallpaper with a map of the world repeated in all directions so that from any point A, if you went a meter up or down, or one meter to either side, the new point B would represent the same spot on the earth as did A. He asked Boris Elegante of a posh Beverly Hills interior decorator firm if this would be a problem.

Boris said he'd just take a map from Rand McNally and repeat it to make the wallpaper. Badde said, "No, no!. That would put the South Pole next to the North Pole. Forget about distances, but I want any path between two points to follow an itinerary my band could *really* cover in a tour.

Boris said he hadn't heard of such wallpaper before, but he'd try. Six months later Boris called to say he couldn't seem to make what Badde wanted. Why not?

Your thoughts on World Wallpaper: Difficulty to either do, or to understand? Worth for asking a vector calculus class? Quality of story? Relation to the curriculum?

Final Words: No taxonomy of *problem solving strategies* gets to the psychological issue of what problems are about. Why are they there? How do you separate exercises from problems? What is reasonable expectation of student performance on problems?

The phrase *problem solving* is at the heart of the difficulty. In its place, I suggest *problem presentation*. It is more alliterative, and it loosens the hold of time on our thinking. A classroom discussion with students taking part in the problem discussion must give them a chance to *respond*. Problems—not exercises, but serious well-stated problems—are the *proverbs* of Mathematics and Science. Good problems are unique. They are worthy of repetition and slow presentation. They are pithy condensations of long lessons. You may use them as good coin in the barter of ideas. The race to problem solving closure smothers the curricular story hidden in a problem.

What makes learning and forgetting a dynamic process: You can't learn most things by going through them once. To be useful learning must be recoverable in many contexts. One can guess that requires more serious use of how long term memories form, and whether the connections to other topics must be created through a learning process. There are many cognitive research hints, from using MRIs and identifying protein pathways.

Yet, as educators, how can we participate? First: Recognize that we are the front line for motivating the research: We have students and they have trouble learning. Even better, we are students, and we often have trouble learning. Sometimes we can see a relation between our problems with learning the same subjects with which our students struggle. Even better, sometimes despite small struggles we learned quite well certain topics with which our students struggle.

One goal of a teacher is to use how we learned to retrieve from memory a small central topic that opened the territory to a whole piece of curriculum.