MATH 4820/5820-3: 1ST PROBLEM SET
3-SPACE LINES AND PLANES AND CONTINUED FRACTIONS

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1. Problem and Grade Guidelines

The problems will appear in the Problem set section of the course website. They are designed to be related to the projects in which you will all participate. I expect there to be three problem sets during the semester, each related to two possible project descriptions, for which you each will get to work on one. Here are some other guidelines for how they will work in a given week.

(1.1a) Monday: I remind the problem set is on the website. You can print them out by going to the website by clicking on the URL in an e-mail to you.
(1.1b) Friday: We can discuss them at the beginning of class. I, or a classmate, can suggest how to approach some problem on which you were stuck.
(1.1c) Monday after, you turn them in.
(1.1d) Participation in Friday discussions will be viewed favorably toward your grade. Work on the 3 problem sets will count as 1/4 of your grade.
(1.1e) Projects work counts 1/2 toward your grade. The final counts 1/4.

Recall, the final will be on topics that arise related to questions during the project presentations of you and your classmates. Here is the rough point distribution.

- **40** points for each problem set. **120** pts total.
- **240** points for the project, divided between the presentation with your team (**120** pts) and your *personal* report on your project (**120** pts).
- **120** points for the Final.
- **40** points for special class contribution.

Potential total points **520**.

2. 3-space lines

In 3-space, $\mathbb{R}^3$, you expect two lines to *determine* a plane if and only if they intersect. In this problem we look at two lines in the same style (parametric) we considered two lines in the plane. Then, we decide if they meet, and if so where. Given $\boldsymbol{w} = (w_x, w_y, w_z)$ and $bv = (v_x, v_y, v_z)$, define the dot product

$$\boldsymbol{w} \cdot \boldsymbol{v} = w_x v_x + w_y v_y + w_z v_z.$$

1. Comparing two lines. 30: Consider two lines

$$L_1 = \{(2,1,3) + t(v_x, v_y, v_z) \mid t \in \mathbb{R}\} \text{ and } L_2 = \{(x_0, y_0, z_0) + s(2,1,0) \mid s \in \mathbb{R}\}.$$

In this problem, you choose values of (v_x, v_y, v_z) and (x_0, y_0, z_0) to construct lines with certain properties.

(2.1a) **3 pts**: For what (v_x, v_y, v_z) is L_1 parallel to L_2?
Ans: $(v_x, v_y, v_z) = \{u(2, 1, 0) \mid u \in \mathbb{R}\}$.

(2.1b) **7 pts**: Suppose $(v_x, v_y, v_z) = (1, 3, 2)$. For what points (x_0, y_0, z_0) is there a plane P that contains both L_1 and L_2?
Ans: If they both lie in a plane, then the two lines must meet. That is the same as there exists (s, t) such that

$$(2, 1, 3) + t(1, 3, 2) = (x_0, y_0, z_0) + s(2, 1, 0); \text{ or}$$
$$(x_0, y_0, z_0) = \{(2, 1, 3) + s(2, 1, 0) - t(1, 3, 2) \mid (s, t) \in \mathbb{R}^2\}.$$

(2.1c) **10 pts**: Continue (2.1b). Find $\boldsymbol{w} = (w_x, w_y, w_z)$ and (x_1, y_1, z_1) so

$$\text{(the plane) } P = \{(x, y, z) \mid \boldsymbol{w} \cdot ((x, y, z) - (x_1, y_1, z_1)) = 0\}$$

contains all points of both L_1 and L_2.
Ans: There are many answers, but you want to choose a point (x_1, y_1, z_1) given by (2.1b) at which the lines meet. Then, consider the points on the plane (x, y, z) as defining a direction from the origin given by $(x, y, z) - (x_1, y_1, z_1)$. Those directions include $(v_x, v_y, v_z) = (2, 1, 0)$ and $(v'_x, v'_y, v'_z) = (1, 3, 2)$. Find $\boldsymbol{w}$ so that $\boldsymbol{w} \cdot (2, 1, 0) = 0$ and $\boldsymbol{w} \cdot (1, 3, 2) = 0$.

3. Continued fractions

Here we solve Prob. 3.4.4 (p. 48) of the text. We showed the continued fraction $[t, t, t, \ldots]$ is of the quadratic irrationality that is the solution of $x^2 - tx - 1 = 0$.

(3.1a) **10 pts**: Imitate the proof for $[t, t, t, \ldots]$ to find the quadratic irrationality with continued fraction $[t_1, t_2, t_1, t_2, \ldots]$ for $t_1, t_2 \in \mathbb{Z}^+$.[1]
Ans: The same procedure gives $x = t_1 + \frac{1}{t_2 + \frac{1}{x}}$, or $x = t_1 + \frac{x}{t_2 x + 1}$, or

$$x(t_2x+1) = t_1(t_2x+1)+x. \text{ So, } x \text{ satisfies } t_2x^2 - t_1t_2x - t_1 = 0.$$
$$\text{Or, } x = \frac{t_1t_2 + \sqrt{(t_1t_2)^2 + 4t_1t_2}}{2t_2}.$$

(3.1b) **10 pts** Find values of t_1 and t_2 in (3.1a) which solve Prob. 3.4.4.
Ans: If we take $t_1 = 2$, $t_2 = 1$, then we get $x = 1+\sqrt{3}$. So, the continued fraction expansion for $\sqrt{3}$ is $[1, 1, 2, 1, 2, 1, \ldots]$.

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[1] Hint: If your formula is correct, it should give the same answer as previously when $t_1 = t_2$.