We have seen the presentation of *Lines in the Curriculum,* so we can put many things together. This is material that goes with it from the book.

What the book is calling a vector: On p. 27, it says a vector in $\mathbf{R}^n$ is a **column** matrix with n rows and 1 column. I said the vector you usually think of writing as $(x_1,\ldots,x_n)$ has now become the transpose (p. 101), or tipped on its side. Another difference: vectors from other courses, written horizontally, have commas separating their entries. Matrices don't have commas in them. So, there is no chance of confusing them if we use them both.

Matrix multiplication: We haven't actually covered matrix multiplication yet, except for this one case (p. 35) where the second matrix is given by a single column vector.

The presentation from Wednesday is on the web site: linalglist-spring19 in the bottom section, called *Extra Course Material*. The *parametric line* is exactly what you expect, say, for the solution space for 2 linear equations in 3 unknowns.

Another topic that came up here: p. 26 the *parallelogram law* of addition of vectors. For $\boldsymbol{a}$, $\boldsymbol{b}$ two sides of a parallelgram with a vertex at the origin, and $\boldsymbol{c}$ its diagonal, how can you express $\boldsymbol{b}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$?

Repeat of problems from Last Friday: *p. 11, Prob. #27:* Gives two equations:

$$x_1 + 3x_2 = b_1 \text{ and } a_{2,1}x_1 + a_{2,2}x_2 = b_2$$

For what values of $a_{2,1}$ and $a_{2,2}$ is this consistent; meaning when is there a solution to the equations? **Hint:** The lines would intersect in a point unless they are parallel. Their intersection is the solution set of the two linear equations. To see the condition for inconsistency, *equate the slopes of the two lines*.

p. 28: Asks essentially the same question but,

it changes $x_1 + 3x_2 = b_1$ to $a_{1,1}x_1 + a_{1,2}x_2 = b_1$.

On Friday, I will review *pivot columns, reduced echelon form* and why, for such matrices having this form, we can give the space of solutions.

We then have two steps left to get through the basics of solving linear equations.

- p. 21: Using row reduction to put a matrix in reduced echelon form.
- Why standard row operations don't change the solution space of a matrix equation.
- Understanding Thm. 4 of p. 37: Figuring precisely the range of a matrix function.

1. Why is each vector $\boldsymbol{a}_i$ in the range of A?
2. Why does $A(\boldsymbol{x})=\boldsymbol{b}$ having solutions mean that $\boldsymbol{b}$ is in the *span* – is a linear combination – of the columns of A?
3. What would you like to know about the pivot columns of an augmented matrix to assure the system is consistent and that it has a unique solution?