Lines in the Curriculum:
A persistent obstruction to Achievement
Connecting Algebra and Geometry

Summary

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    - I.A: There are many lines in the curriculum
    - I.B: The Point of Lines: Direction and Orientation
    - I.C: What is a line?: A Persistent Curricular Problem

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   - I.C: What is a line?: A Persistent Curricular Problem
3. **Part II**: How do we use lines to understand solutions of Linear Equations. A presentation for later.

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[In practice the material is usually taught earlier than given here.]

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5. In 11th-12th grade? Answer: An expression like $y = mx + b$.

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3. In 2nd year Calculus or in physics? Answer:

$$\text{In } \mathbb{R}^3 : \{(x_0, y_0, z_0) + t(u, v, w) \mid t \in \mathbb{R}\}.$$

A *parametric line*: t is a *parameter*.

I.B: The Point of Lines: Direction and Orientation

Suppose I want to guide you to somewhere where you are walking across a field. I might use time and direction.

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- How would I explain this to you? Answer: I might use my left arm to point. Maybe you are on an airplane (or a proton) and I'm directing you to a target up in the air.

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- What picture might I have?: Answer: At time $t = 0$ you are at (x_0, y_0, z_0). You end up in one minute at $(x_0 + u, y_0 + v, z_0 + z)$, in 2 minutes at $(x_0 + 2u, y_0 + 2v, z_0 + 2z)$, 3 minutes, etc.

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1. Why would you think this is a line?
2. Answer: It looks like the formula for a line, and it has the right number of variables.
3. Is there a reason it might not be a line?

Using the principle: Two points determine a line.

1. Setting $x = 0$ confines points *on* S to a coordinate plane.

$$S_{x=0} = \{(0, y, z) \mid z = 3y + 4\}.$$

Same with $y = 0$. Do the points so confined look like lines?

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2. In each coordinate plane there is a traditional 9th grade line. Two distinct (not parallel) lines on S meet at $(0.0.4) \in S$.

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3. The points $(1, 1, 9)$ and $(-1, -1, -1)$ are on S. If you have a plane in $\mathbb{R}^2$, would you expect that all points on the line

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they determine are also on S. How would you check? Hint: How would you write expressions for those points?

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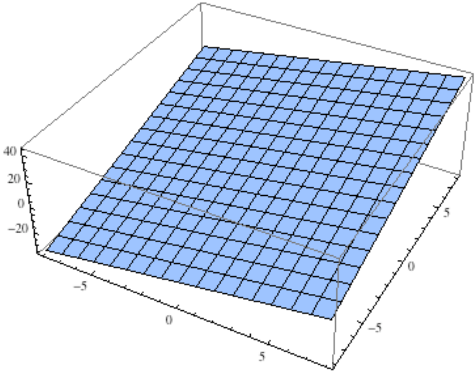
4. Euclid had no definition for a *plane*: No joke!

Mathematica produces S

Two lines on a Plane



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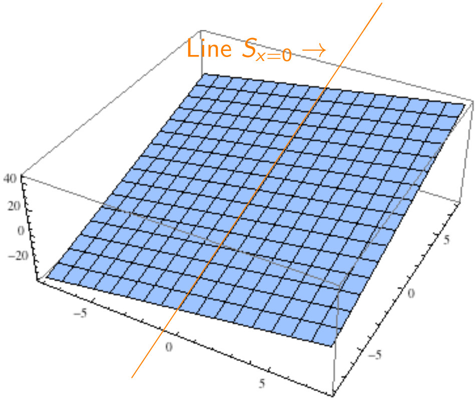

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