Alumni Voices

Algebraic Equations and Finite Simple Groups: What I learned from graduate school at the University of Michigan, 1964–1967, by Michael Fried

Michael Fried (PhD 1967) *spent 26 years as a member of the Mathematics faculty at the University of California, Irvine. During his career, he received a Fulbright Fellowship and a Humboldt Award. He has contributed this story as part of our ongoing history project, and an expanded version of the article is available on our History webpage www.math.lsa.umich.edu/information/depthistory.shtml. The full detail of this story is available on Fried's webpage: www.math.uci.edu/~mfried/paplist-cov/UMStory.html.*

PRELUDE: After three years of Graduate School in Mathematics at University of Michigan (1964-1967), writing a thesis under the direction of Don Lewis, I left for a postdoctoral at the Institute for Advanced Study. There I studied with Goro Shimura. My first year was extended to two years (1967-1969). Then, I went with James Ax to SUNY at Stony Brook. After receiving tenure and a Sloan Fellowship, I left. That bare bones outline of a beginning career tells little mathematically.

It has no hint that the work inspired by my time at UM connected resolutely with the simple group classification—through conversations with John Thompson—and with modular curves—interactions with J.P. Serre. Nor, that problems of Andrzej Schinzel and Harold Davenport, (visitors to UM my second year) in papers with Lewis, were the inspiration. Not even that technical tools came from assiduous use of Grothendieck's fiber product approach to algebraic equations. Yet, fulfilling those connections required–no word better–tutoring from many UM-affiliated faculty during my formative years.

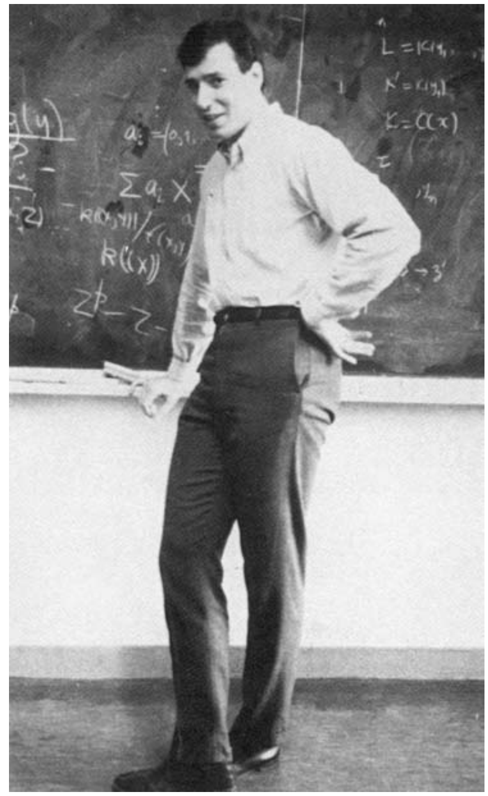
This short version of the story relates Davenport's Problem, the steps in its solution, and how the connections above came about. I never lost my youthful enthusiasm for completing programs of Abel, Galois and Riemann, as recorded in "What Gauss told Riemann about Abel's Theorem" (http://www.math.uci.edu/~mfried/paplist-cov/Wh-Gauss-Tld-Riem-ab-Abel.pdf).

DAVENPORT'S PROBLEM AND FIBER PRODUCTS: When number theorists say almost all primes p, they mean all but finitely many. Davenport sought relations between two polynomials $f(x)$ and $g(y)$ with rational coefficients—where no change of the variable in f gives g—having the same ranges on integers mod p for almost all p. He liked this style of question, and often used exponential sums to interpret it.

Changing "almost all" to "infinitely many" and taking $g(y)=y$, restates the hypothesis of Schur's 1921 conjecture. The conclusion of Schur: f must be composed of linear, cycle and Chebychev polynomials. Richard Brauer was a student of Schur, and advisor of Don Lewis. When I met him he asked if I knew he had worked on Schur's conjecture. I hadn't.

A variables-separated algebraic equation looks like $f(x)$-$g(y)$=0. Writing this as $f(x)$-z=0 and $g(y)$-z=0 opened the territory. Fiber products of f and g over the z-line allowed me to use groups to draw conclusions. I'll use n for the degree of f. Visiting Assistant Professors Armand Brumer and Richard Bumby guided my mastering Grothendieck's Tohoku paper and pieces of his EGA. There I learned to go between algebraic equations and group theory.

Chuck MacCluer's thesis, under Lewis, showed—for special f—a geometric statement gave Schur's one-one mapping hypothesis. Later, by extending the Chebotarev density theorem, I formulated a general context including Davenport and Schur. There, pure group theory translated the number theory.



Michael Fried, circa 1968. Photo courtesy of the American Mathematical Society.

Formulations, however, are not conclusions. Could you invert the direction polynomials to groups, as in Schur's Problem?

A rational function not composed of lower degree functions is indecomposable. I state lightly what I found powerful in practice: (i) A rational function's covering group is primitive exactly when the function is indecomposable; (ii) A polynomial's covering group always contains an n-cycle.

From that I learned, that if f was indecomposable, its covering group was either doubly transitive, or f was in the Schur conclusion. That finished the Schur story and deepened the Davenport story.

I now knew Davenport's hypothesis on f and g produced a difference set mod n that encoded how zeros of $g(y)$-z=0 summed to a zero of $f(x)$-z. Not only did the distinct permutation representations for f and g have the same degree, their group representations were identical.

I had seen Brumer pepper Jack McLaughlin with group theory questions. When Brumer left for Columbia in 1966, I took his place with McLaughlin, from whom I learned the distinction between doubly transitive and primitive. Richard Misera, a fellow grad student working with Donald Higman, saw this interaction and gave me a propitious example, coming from projective linear groups. I applied this, modulo something that I learned very much on my own—R(iemann)'s E(xistence) T(heorem)—to produce polynomial pairs having almost simple groups with special projective linear core. The three propitious points were these:

1. Without writing equations, I was able to see the Galois action of the cyclotomic field of n-th roots of 1, acted on the difference set relating f and g. The elements that preserved that difference set, up to translation (so-called multipliers), gave the definition field of the pair (f,g). Further, -1 was never a multiplier, so that definition field was never Q.

2. Because the covers given by f and g had genus 0, the only possible degrees for f and g were n=7, 11, 13, 15, 21, 31.

3. The cases with infinitely many essential pairs (f,g) modulo mobius action on z, x and y appearing in #2, had degree 7, 13 and 15. Further, in these cases those essential parameters formed a genus 0, upper half-plane quotient, that wasn't a modular curve.

Tom Storer, newly at UM when I visited it from IAS, worked with me on the last statement of #1. This completed Davenport's Problem over Q for indecomposable polynomials f. There were no nontrivial examples. It used nothing from the simple group classification. The offshoot of that technique became the Branch Cycle Lemma, the most practical tool by which to relate geometric covering groups and definition fields.

The longer version of this article has these further sections: Using the classification and the genus 0 problem; UM seminars and modular curves; The significance of Davenport's problem. These tell why Davenport's problem had an impact (in order) in well-known papers of Thompson, Serre, and Denef and Loeser.

Of the others who got PhDs in 1967, one became much more famous than anyone who might be reading this. That was "The Unabomber," a no-show at the going away party Paul Halmos gave us. The picture on the opposite page is from Halmos' "I Have a Photographic Memory" (American Mathematical Society, Providence, RI, 1987). I was standing in front of my diagram for the Schur Conjecture proof at the end of my 1968 UM lecture on it.

I didn't know about that picture until years later. Still, either I, or the Schur Conjecture, must have been funny. A New Yorker not long afterwards had a cartoon based on it. I have seen only one person from my graduate years more than once after grad school. That was the topologist Bob Edwards who twice sat in on talks of mine at AMS conferences.

Solution to Math Problem

There will be a final string of beans of the same color (the color of the last bean drawn), and then a bean preceding it of a different color. The desired probability p is that these two colors are blue and green or green and blue, respectively. The probability that the last bean drawn is blue is 300/600 = 1/2, and, no matter how long the final string of blue beans is, the probability that the bean of preceding color is green is 200/(100+200) = 2/3. For green and blue these numbers become 200/600 = 1/3 and 300/(100 + 300) = 3/4. Thus, p = (1/2)(2/3)+(1/3)(3/4) = (1/3)+(1/4) = 7/12.

Share Your Stories

We would like to include your stories and remembrances on our History website www.math.lsa.umich.edu/information/depthistory.shtml. We have recently begun in earnest to collect and share some of our Department's history. Over the years many documents have been written about the educational, scholarly, administrative and research activities of U-M mathematics, and we invite you to share the history.

Beyond the facts, figures and remembrances of faculty members, the history of the Department is contained in its students. We are interested in your memories of your time as a student. What impressed you about your first or hardest math class? Did a discussion during a seminar help to establish the direction of your significant research? Who were your most memorable instructors? Do you have a story about one of the faculty members that might enlighten others to their spirit? How did they help shape your educational career and influence your life? Do you have a story about Tom Storer, Maxwell Reade, T. H. Hildebrandt, George Piranian, or any other Department member that stands out in your memory?

Please send us your remembrances and we will make an effort to share them with others. If you would like your remembrances to be included in ContinuUM or on our website, we will work with you to include your memories.

Michigan Today is also seeking your stories. You can share your stories and read those of others on this website http://michigantoday.umich.edu/heritage.php.

We Need You!

Want to get involved with the UM Department of Mathematics? Here are some areas where alumni participation is vital. Contact us if you are interested in working with us on these initiatives.

- Recommend the UM mathematics program to students interested in undergraduate or graduate studies
- Participate in our annual Career Day, held each year in early November
- Visit the Department for afternoon tea (weekdays at 3:45 sharp) if you are in town for the weekend, including Homecoming, Parent's weekend, or the Presidential Society weekend
- Be a mentor (in person or via email) to a current student
- Set up a recruiting program with your company for graduating students
- Offer internships in your company to mathematics students
- Allow groups of mathematics students to visit your company
- Give an informal talk to mathematics students about how you have used your math knowledge

Email math.mich@umich.edu
or call 734-647-4462