MATH 120A SAMPLE MIDTERM EXAM

WINTER 2015

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- When time is called, you must stop working immediately, close your exam, and remain seated until your exam is collected.
- If you want to leave your seat for any reason before time is called, raise your hand and remain seated until acknowledged.

1	/ 10
2	/ 4
3	/ 4
4	/ 4
5	/ 4
6	/ 4
Total	/ 30

Problem 1 (10 points). Mark each statement 'T' for true (meaning always true) or 'F' for false (meaning sometimes false). You do NOT need to justify your answers to this problem.

T F The operation $*$ on $\mathbb{R}$ defined by $a * b = \max(a, b)$ is associative.
T F The set of all nonzero integers forms a group under multiplication.
T F Any two groups of order 4 are isomorphic to each other.
T F Every element of the group $(\mathbb{Z}, +)$ has infinite order.
T F Let A be a set. If there is a surjection from $\mathbb{N}$ to A, but there is no bijection from $\mathbb{N}$ to A, then A is finite.

T F Let $(S, *)$ be a binary operation and let $A, B \subseteq S$. If A and B are closed under $*$, then their intersection $A \cap B$ is closed under $*$.
T F If a and b are identity elements of a binary structure $(S, *)$, then $a = b$.
T F The group $(\mathbb{R}^*, \cdot)$ of nonzero real numbers under multiplication has a subgroup that is isomorphic to $\mathbb{Z}_4$.
T F If G is a cyclic group, then G has no subgroups other than $\{e_G\}$ and G.
T F Let G_1 and G_2 be subgroups of $GL_2(\mathbb{R})$ under matrix multiplication. If every element of G_1 is diagonal, and $G_1 \cong G_2$, then every element of G_2 is diagonal.

Problem 2 (4 points). Let S be a set.

(a) What is a *binary operation* on S?

(b) If $*$ is a binary operation on S and A is a subset of S, what does it mean for A to be *closed* under $*$?

Problem 3 (4 points). Let G be a group and let $a \in G$. Define the function $\phi : G \to G$ by $\phi(x) = axa^{-1}$. Prove that ϕ is an isomorphism from G to G.

Problem 4 (4 points). Let G denote an arbitrary group. Prove that the property P saying

$a^2 = e_G$ for every $a \in G$

is a structural property.

Problem 5 (4 points). For each part, make sure to justify your answer:

(a) Give an example of a set S and an operation $*$ on S that is associative but not commutative.

(b) Give an example of a set S and an operation $*$ on S that is commutative but not associative.

Problem 6 (4 points). Consider the set of matrices $G = \{I, A, B, C\}$ where

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

Then G forms a group under matrix multiplication (you may assume this.)

(a) Compute the order of each element of this group G.

(b) Draw a subgroup diagram for G.