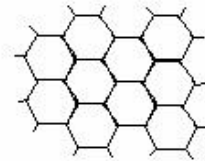
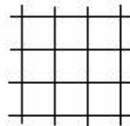
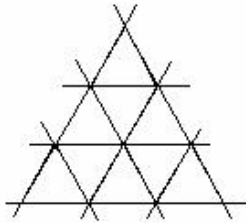


Exercises for January 16, 2010

1. In each of the three cases below, what is the minimal number of colors to be used if we want to color the cells in such a way that any two with a common side have different colors?



2. What is the maximal number of rooks which one can place on a chess board in such a way that no two rooks would attack each other?
3. The upper drawer of a drawer chest has 5 identical white gloves for the right hand and 10 identical white gloves for the left hand. The lower drawer has 10 identical black gloves for the right hand and 15 identical black gloves for the left hand. The contents of the drawers was mixed and put in a big non-transparent plastic bag. What is the minimal number of gloves that one should pull out of the bag without looking into it, to make sure that one ends up with at least one valid pair of gloves (same color, one left and one right)?
4. The field for playing the Battleship game is an 8x8 square. Right now it has a single ship shaped like a 1x4 rectangle. What is the minimal number of times one should fire in order to (a) damage the ship (b) identify its position completely?
5. A medieval city has 6 streets. To protect the city its citizens want to build a wall around it. The wall is supposed to have a shape of a polygon and each of the walls should be parallel to one of the streets. On each of its corners the wall will have a tower. What is the maximal possible number of towers? Draw a picture?
6. A party had 10 guests and all came in the same model of flip-flops, although all 10 had different sizes. In the evening the guests started leaving, putting on any pair of flip-flops they could (i.e. any pair which is still left and has the size equal to at least that of the guest's). Assume that no pair of flip-flops was separated. At some moment it turned out that no further guests can leave since the remaining pairs are too small. What is the maximal number of guests at that moment?
7. To put the glass in 15 windows of different rectangular) shapes and sizes, 15 pieces of glass were cut to fit each of the windows exactly. A worker does not know that the pieces had been fit to the windows, and he uses the following strategy: he goes to the next window with all unused pieces of glass and goes through the pieces until he finds one which is large enough (i.e. fits perfectly or larger in both directions, so that he can cut it down to size). If he cannot find such a piece, he goes to the next window. One is not allowed to paste several pieces together. What is the maximal number of windows which can left without a glass?
8. On a joint conference of liars and truth lovers, 32 participants were elected to the Managing Board. They were arranged into 4 rows with 8 people each. During a

discussion break each of them said: "Among my neighbors, there are representatives of both parties." What is the minimal possible number of liars on the board? (A neighbor can be sitting to the left, to the right, in front or behind).

9. A map show several countries of rectangular shape. John want to color the map in such a way that any two countries with a common part of their border (more than just a corner) have different colors. What is the minimal possible number of countries on the map if it is known that three colors are not enough?