

Exercises for November 14, 2009

1. A chess tournament had six participants. Each pair of participants played a game. What is the total number of games played? How many games were played by each participant? If a winning player gets 2 points, a losing player 0 points and a draw gives 1 point to each of the players, what was the total sum of the points of all participants?
2. A boy said to his friend: "There are 35 students in our class. And - can you imagine? - each of them has exactly 11 friends in the class". Is that possible?
3. The Small City has 15 telephone booths. Can you connect the booths by wire in such a way that 4 of the booths are connected with three others, 8 booths are connected with six others and 3 booths with five others?
4. Prove that if N is the number of people on Earth which made an odd number of handshakes, then N is even.
5. In a group of seven boys, each has at least three brothers among the remaining boys. Prove that in fact all seven must be brothers.
6. The Small City still has 15 telephone booths, and each of them is now connected by a wire with at least 7 other booths. Prove that if each booth has a person with a unique gossip tale, then all of them can learn all the available gossip (a gossip tale may travel from booth A to booth B through some of other booths, as long as they are connected with the wire).
7. On a dancing party each boy has danced with exactly 10 girls while each girl with exactly 9 boys. What was greater - the number of boys or the number of girls?

Homework

1. There are 50 scientists at a conference and each knows at least 25 other participants of the conference. Prove that it is possible to find 4 of them so that, when they sit down at a round table, each of them will know both of his neighbors.
2. A family of 24 mice lives in a mouse hole. Every night four of them walk to the food store to steal some cheese. Can it happen that at some moment in time each of the mice has visited the store exactly once?
3. Each of the 102 students in a school knows at least 68 other students. Prove that there will be at least four of them who know the same number of people.