

### Exercises for October 24, 2009

1. In a square 1 m. by 1 m. someone marked 51 dots. Prove that at least three of those can be covered by a smaller square 20 cm. by 20 cm.
2. A room has a shape of a regular hexagon with the side 3m. In each of the six corners there is a special device, a "snore-meter" which detects how many students are snoring within 3m from it. The total sum of the numbers shown by all six devices, is equal to 7. How many students are snoring in that room?
3. 17 chess kings are placed on the chess board. Prove that at least two of them are attacking each other.
4. The Parliament of a country has 450 members. During an emotional discussion, each of them had slapped exactly one of his colleagues in the face. The next day they need to elect a parliament committee which will consider an entirely different matter. Show that this can be done in such a way, that no member of the committee had previously slapped any other member of the committee.
5. Several straight lines divide the plane in some number of pieces. Show that these pieces can be colored blue or red, so that any two pieces with common border will be painted in different colors.
6. You work at a store which has received a container with 300 boots in it. It is known that 150 boots are for the left foot and 150 are for the right foot, but they all come in three sizes: M, L and XL. Show that it is possible to find at least 50 valid pairs (i.e. both of the same size, one of the left, another right).

### Homework

1. Numbers 1, 2, 3, ..., 20 are written on the board. One is allowed to do the following operation: erase any two numbers A and B and replace them with a single number  $AB + A + B$ . What will be left after 19 such operations?
2. What is the maximal number of squares in an 8x8 board which can be painted black, so that each L-shaped corner of 3 squares will have at least one unpainted square? What is the minimal number of squares in an 8x8 board which can be painted black, so that each L-shaped corner of 3 squares will have at least one black square?