ONE-SIDED GORDONIAN RECTANGLES

Once I got started on Gordonian Rectangles, the ideas just kept coming. Here's another one:

Example 20

					3		2	
			6	4				
	3	5		7		8		1
	9				7			2
3		2				1		9
1			2				7	
2		4		9		3	6	
				3	4			
	8		7					

Without Gordonian Logic, you stop here:

Example 20-1

478	14	179	158	58	3	79	2	6
78	2	179	6	4	18	79	3	5
6	3	5	9	7	2	8	4	1
45	9	68	3	1	7	46	58	2
3	7	2	4	6	58	1	58	9
1	45	68	2	58	9	46	7	3
2	15	4	158	9	158	3	6	7
57	6	17	15	3	4	2	9	8
9	8	3	7	2	6	5	1	4

A One-Sided Gordonian Rectangle occurs when you have four cells in two different boxes where two adjacent sides have the same two candidates and the other two sides have those two candidates plus the same third candidate. Cells 13, 23, 17, and 27 form one. If we assume that cells 13 and 23 both do not contain a 1, that would result in two valid solutions, since we could have either 7's in cells 13 and 27 and 9's in cells 23 and 17, or 9's in cells 13 and 27 and 7's in cells 23 and 17. Since we know that there is one solution only, we know that either cell 13 or 23 has to have a 1 in it. And that tells us that cell 83 cannot contain a 1, so it must be a 7. Once you place that 7, the rest of the puzzle is a walk in the park.

Example 20 Answer

7	4	1	8	5	3	9	2	6
8	2	9	6	4	1	7	3	5
6	3	5	9	7	2	8	4	1
4	9	8	3	1	7	6	5	2
3	7	2	4	6	5	1	8	9
1	5	6	2	8	9	4	7	3
2	1	4	5	9	8	3	6	7
5	6	7	1	3	4	2	9	8
9	8	3	7	2	6	5	1	4

GORDONIAN POLYGONS

What works for rectangles also works for figures of more than four sides. The idea for these came to me while walking home from work after spending too much time on sudoku books.

Example 21

		8						
	6		8		4	9	3	
4					2	1		
2	9	3				4	5	
				4				
	4	1				3	6	9
		6	2					3
	8	4	5		9		2	
						8		

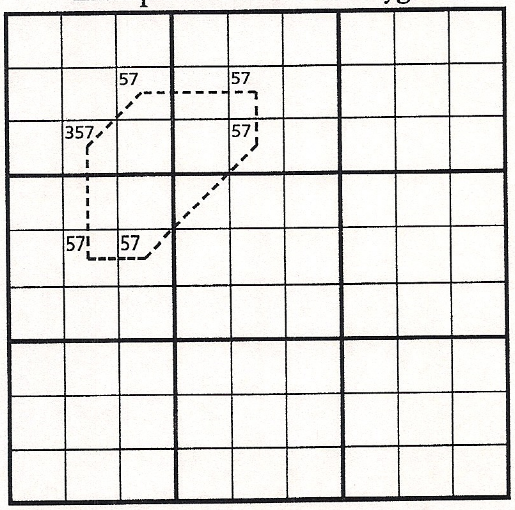
Using all of our standard methods gets us to this point:

Example 21-1

35	2	8	136	9	16	56	7	4
1	6	57	8	57	4	9	3	2
4	357	9	36	57	2	1	8	56
2	9	3	16	168	168	4	5	7
6	57	57	9	4	3	2	1	8
8	4	1	7	2	5	3	6	9
9	15	6	2	18	178	57	4	3
37	8	4	5	136	9	67	2	16
357	135	2	4	136	167	8	9	156

Check out the contents of cells 23, 25, 32, 35, 52, and 53. Each cell contains just the candidates 5 and 7, except for cell 32, which has 3 as an additional candidate. These cells form a polygon as shown below in which the vertices are in pairs in each row, column, and box. There are exactly two vertices in rows 2, 3, and 5, exactly two vertices in columns 2, 3, and 5, and exactly two vertices in boxes 1, 2, and 4. If cell 32 weren't a 3, there would be no way to ever determine whether those six cells are (going clockwise from cell 32) 5-7-5-7-5-7 or 7-5-7-5-7-5. So cell 32 must be a 3.

Example 21 Gordonian Polygon



With the 3 in cell 32, finishing up is a snap.

Example 21 Answer

5	2	8	3	9	1	6	7	4
1	6	7	8	5	4	9	3	2
4	3	9	6	7	2	1	8	5
2	9	3	1	6	8	4	5	7
6	7	5	9	4	3	2	1	8
8	4	1	7	2	5	3	6	9
9	1	6	2	8	7	5	4	3
3	8	4	5	1	9	7	2	6
7	5	2	4	3	6	8	9	1

GORDONIAN POLYGONS PLUS

Just as Gordonian Rectangles led to Gordonian Rectangles Plus, Gordonian Polygons leads to Gordonian Polygons Plus. In these you have a polygon where all but one vertex has two candidates, and the last vertex has those same two candidates plus two or more others. You can narrow down the possibilities of the last vertex to the candidates that aren't shared with the other vertices. As always, an example helps.

Example 22

	2							
4					9			5
	8		3		5			2
9				8		1		
5	3		7		1		6	8
		6		5				9
6			9		4		8	
3			1					7
							4	

Things appear to be stuck at this point:

Example 22-1

17	2	5	8	1467	67	3469	39	346
4	6	3	2	17	9	8	17	5
17	8	9	3	46	5	46	17	2
9	47	247	6	8	23	1	5	34
5	3	24	7	9	1	24	6	8
8	1	6	4	5	23	7	23	9
6	57	17	9	237	4	235	8	13
3	459	48	1	26	68	2569	29	7
2	79	178	5	367	678	369	4	136

To find the Gordonian Polygon, look for a pair of candidates that appears several times. If you can connect them with one cell that has those two candidates plus at least one other, and the vertices are in pairs in all the rows, columns, and boxes, then you've got it. Here, all those cells with candidates 1 and 7 stand out like a sore thumb. Connecting cells 11, 31, 38, 28, and 25 with the key cell, 15, creates a Gordonian Polygon Plus, since cell 15 has more than three candidates. We can eliminate candidates 1 and 7 from cell 15, leaving 4 and 6 as candidates, which form a pair with cell 35. Because of that pair, the 6 in column 5 will be in cell 15 or 35, so a 6 can't be in cell 85, which means cell 85 has to be a 2. Put the 2 in and the rest is duck soup.

Example 22 Answer

1	2	5	8	6	7	9	3	4
4	6	3	2	1	9	8	7	5
7	8	9	3	4	5	6	1	2
9	7	4	6	8	2	1	5	3
5	3	2	7	9	1	4	6	8
8	1	6	4	5	3	7	2	9
6	5	7	9	3	4	2	8	1
3	4	8	1	2	6	5	9	7
2	9	1	5	7	8	3	4	6

ONE-SIDED GORDONIAN POLYGONS

You had to know it was coming.

Example 23

	6			8				
		9	4	5			2	
	1		7			4	3	
		6						2
		7		2		6		
8						7		
	9	2			5		6	
	7			6	4	8		
				7			4	

Here's the point at which Gordonian Logic is needed to continue:

Example 23-1

23	6	4	13	8	123	59	7	59
7	38	9	4	5	36	1	2	68
25	1	58	7	9	26	4	3	68
1359	345	6	159	34	7	59	8	2
1359	345	7	159	2	8	6	159	34
8	2	15	6	34	19	7	159	34
4	9	2	8	1	5	3	6	7
15	7	3	2	6	4	8	59	159
6	58	158	39	7	39	2	4	15

The One-Sided Gordonian Polygon is at cells 42, 45, 52, 59, 65, and 69. It's just like a Gordonian Polygon except that two adjacent vertices have three candidates. In this case, they're cells 42 and 52. One of those two must be a 5, so we know that cell 92 cannot be a 5, and thus must be an 8. That gives you a green light to speed ahead to the finish.

Example 23 Answer

2	6	4	3	8	1	5	7	9
7	3	9	4	5	6	1	2	8
5	1	8	7	9	2	4	3	6
3	5	6	1	4	7	9	8	2
9	4	7	5	2	8	6	1	3
8	2	1	6	3	9	7	5	4
4	9	2	8	1	5	3	6	7
1	7	3	2	6	4	8	9	5
6	8	5	9	7	3	2	4	1

STILL MORE GORDONIAN SHAPES

Once you understand the concept of Gordonian Logic, you can use it whenever you encounter a situation where there is a potential for two solutions. Since you know there can't be two solutions, you can choose the candidate that prevents it. Here are two examples.

EXTENDED GORDONIAN RECTANGLES

This variant on the Gordonian Rectangle was discovered by Francis Heaney, and is sometimes called a Franciscan Rectangle. Here's the situation in a nutshell:

Extended Gordonian Rectangle

		23			12			13
		23			12			134

Here we know that the cell with three candidates must be a 4. If it weren't a 4, there would be two valid solutions, as shown in the next column, and we know there is one solution only, so we know a 1 or 3 in the "134" cell must lead to an impossibility.

Extended Gordonian Rectangle Possibilities

		3			2			1
		2			1			3

		2			1			3
		3			2			1

Of course, this type of Gordonian Rectangle can be one-sided, as well. If the cell with candidates 1 and 3 instead had the candidates 1, 3, and 4, we'd know that the 4 had to be somewhere in one of the two cells with 1, 3, and 4, and we could eliminate 4 as a candidate from other cells in that column.

Below is an example of a puzzle that requires the use of an Extended Gordonian Rectangle.

Example 24

3						4		
	1	4		6	7			
		9			2		1	6
	3				8	9	4	
				5				
	9	2	6				7	
6	2		9			7		
			2	7		6	8	
		8						2

We get halted in our tracks at this point:

Example 24-1

3	6	5	8	9	1	4	2	7
2	1	4	35	6	7	8	35	9
78	78	9	345	34	2	35	1	6
1	3	6	7	2	8	9	4	5
48	48	7	1	5	9	2	6	3
5	9	2	6	34	34	1	7	8
6	2	13	9	8	345	7	35	14
9	45	13	2	7	345	6	8	14
47	457	8	34	1	6	35	9	2

Looking at cells 31, 32, 51, 52, 91, and 92, we know that if cell 92 were not a 7, then there would be two valid ways to arrange the numbers 4, 7, and 8 in those six cells, so we can conclude that cell 92 must be a 5. Once you have that 5, it's a cinch to finish. (In this same diagram is an unhelpful One-Sided Extended Gordonian Rectangle, in cells 73, 83, 76, 86, 79, and 89. We know that either cell 76 or 86 is a 5, but that gets us nowhere.)

Example 24 Answer

3	6	5	8	9	1	4	2	7
2	1	4	5	6	7	8	3	9
8	7	9	3	4	2	5	1	6
1	3	6	7	2	8	9	4	5
4	8	7	1	5	9	2	6	3
5	9	2	6	3	4	1	7	8
6	2	1	9	8	3	7	5	4
9	4	3	2	7	5	6	8	1
7	5	8	4	1	6	3	9	2

GORDONIAN RECTANGLE WING

This is sort of a combination between a Gordonian Rectangle and an XY-wing, which you will learn about in Chapter 9. Suppose you come to a situation like this:

Gordonian Rectangle Wing

		45					34	
		34				123	12	
						12	123	

Cells 47, 48, 87, and 88 form a kind of Gordonian Rectangle. We know for certain that a 3 has to be in either cell 47 or cell 88. If cell 47 is a 3, then cell 43 would have to be a 4, which means that cell 13 would be a 5. And if cell 88 is a 3, then cell 18 would have to be a 4, and again that means that cell 13 would be a 5. So while we don't know if it's cell 47 or 88 that is the 3, we know that one of them has to be a 3, and no matter which of them is a 3, the result is the same: Cell 13 is a 5.

Naturally, the same kind of situation can arise with Gordonian Polygons. The possibilities are almost endless. For example, in the above diagram, if cell 58, 68, 77, or 97 had 3's in its candidate list, the 3 could be removed, since having a 3 in any of those cells would create two solutions in cells 47, 48, 87, and 88.