Dominant Strategies (From Last Time)

- Continue eliminating dominated strategies for B and A until you narrow down how the game is actually played. What strategies should A and B choose? How are these the "best" strategies?

	B1	B3
A1	(4,3)	(6,2)
A2	(2,1)	(3,6)
A3	(3,0)	(2,8)

- This is called the iterated elimination of strictly dominated strategies. What happens if we apply this method to the Prisoner's dilemma?

- The following game is called a stag hunt:

Stag Hunt

	B1	B2
A1	$(2, 2)$	$(0, 1)$
A2	$(1, 0)$	$(1, 1)$

Are there strictly dominating strategies in this game?

- Suppose that two crooks are meant to be playing a Prisoner's dilemma, but they are very prideful. In particular, not betraying is worth 4 years in prison to each of them. How does the game change? Write out the normal form.

	B1	B2
A1		
A2		

Does this game feature strictly dominating strategies?

- Recall the conflict between Sony and Toshiba. If Sony chooses first, are there strictly dominating strategies? What are they, and what is the outcome of the game if both players are perfectly rational.

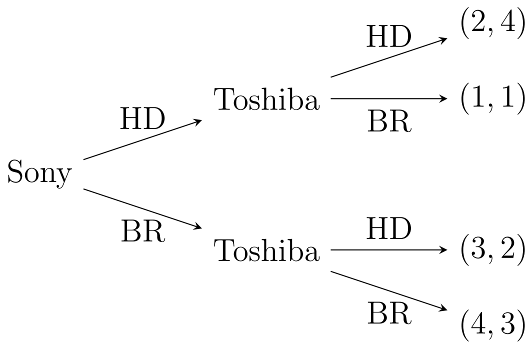
- The more realistic situation is that Sony was in a better position than Toshiba. Let's suppose that Sony is confident that if they use Blue Ray and Toshiba uses HD, Blue Ray will eventually catch on. We suppose Sony feels this way because of their use of Blue Ray in the PS3 and their successful marketing campaign. Sony's preferences and Toshiba's are different, so that the situation (if they made their decisions simultaneously) would be as follows.

	B1 (HD)	B2 (BR)
A1 (HD)	(2,4)	(1,1)
A2 (BR)	(3,2)	(4,3)

Extensive Form

Recall that a strategy for a player in a game must completely describe their actions under any possible scenario. When games are played with two players taking turns, it can be difficult to completely describe a strategy. A different way of representing a game where players take turns in the extensive form.

Consider Sony and Toshiba again, where as on the previous page Sony is in a better position than Toshiba. Let's assume Sony picks first, and Toshiba can then make a decision based on what Sony does.



Toshiba's strategies are more complicated than simply picking what to do, because they have to consider all the possible choices that Sony could have made. One strategy for Toshiba would be something like: if Sony picks HD I'll pick HD, and if Sony picks BR I'll pick BR.

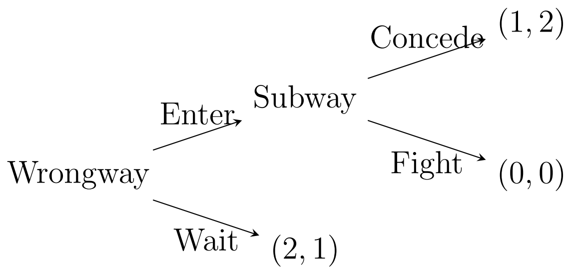
Sony still has only two strategies: pick HD or pick BR. We can describe Toshiba's strategies by talking about their two choices: the first choice is what they do if Sony chooses HD, the second is what they do if Sony chooses BR.

Fill out the following normal form of the game that Sony and Toshiba are playing.

	B1 (HD/HD)	B2 (HD/BR)	B3 (BR/HD)	B4 (BR/BR)
A1 (HD)				
A2 (BR)				

- Using the previous table, eliminate rows and columns using strictly dominating strategies. What is the outcome of the game?

- Suppose that Subway (player A) is a well established sandwich restaurant that doesn't like competition. You and your friends have a bunch of money and are considering starting a competing sandwich company Wrongway. You play the following game, where you decide whether to spend your money developing the company (entering the market, or waiting and keeping your money). If you do develop it, Subway can either concede (C) and allow you to take some of their profits away, or they can fight you (F) which takes up their resources but also harms your business.



Write out the normal form of the game.

	B1 (Enter)	B2 (Wait)
A1 (C)		
A2 (F)		

Are there dominating strategies?

Mixed Strategies

We say that a strategy is pure if it is one of the original options in the game. On the other hand, what is there to stop a player from randomly picking one of their possible strategies? I new strategy built by randomly choosing other strategies is called a mixed strategy.

Consider Rock-Paper-Scissors. The pure strategies are Rock, Paper, and Scissors, meaning a strategy in which you have picked (forever and always) one of those three options. The options have the following payouts (we assume you're playing for $1).

	B Rock	B Paper	B Scissors
A Rock	(0,0)	(-1,1)	(1,-1)
A Paper	(1,-1)	(0,0)	(-1,1)
A Scissors	(-1,1)	(1,-1)	(0,0)

There are no strictly dominated strategies, and clearly none of the three options is "the best".

Most players have an idea that the "best" way to play the game is to randomly pick from all three strategies. If you tend to play one strategy over another then you could be exploited by someone who favors the strategy that beats it. How do we make this precise?

Let's experiment with a simpler game. Let's say that Subway and Wrongway are competing as in the last exercise.

	B1 (Enter)	B2 (Wait)	
A1 (C)	(1,2)	(2,1)	p
A2 (F)	(0,0)	(2,1)	1-p
	q	1-q	

We want to assign probabilities to each of the two options for each player. We say that Subway plays (C) (concede) with probability $0 \leq p \leq 1$, and (F) (fight) with probability $1 - p$. Similarly, Wrongway plays Enter with probability $0 \leq q \leq 1$ and plays Wait with probability $1 - q$.

What should a player be trying to do when they randomize their strategies? Well, if a Subway's random strategy makes one of the options better for Wrongway, then Wrongway will certainly play that strategy in order to exploit their advantage. The only way that Subway can stop this from happening is by making both choices for Wrongway equally important.

Consider the following augmented matrix.

	B1 (Enter)	B2 (Wait)	
A1 (C)	(1,2)	(2,1)	1q + 2(1-q) = 2-q
A2 (F)	(0,0)	(2,1)	0q + 2(1-q) = 2 - 2q
	2p + 0(1-p) = 2p	1p + 1(1-p) = 1	

Below and to the right of all the choices we show what the payoff should be for Subway and Wrongway if they choose any of their options. If Wrongway chooses to Enter, then Subway will choose to Concede with probability p and to Fight with probability $1-p$. This means the payoff for Wrongway would be 2 p-amount of the time, and it will be 0 $1-p$ amount of time. They expect a payoff of $2p + 0(1-p) = 2p$. On the other hand, if Wrongway chooses to Wait then their expected payoff is $1p + 1(1-p) = 1$.

In order for Subway to play the game well, they should choose p so that both options are equally beneficial for Wrongway. This amounts to solving

$$2p = 1$$

So that $p = 0.5$. Then Subway's decision is (before the game starts) to either Concede or Fight half of the time each, randomly.

Similarly, Wrongway has to equalize the two sides from Subway's perspective. This means solving

$$2 - q = 2 - 2q$$

so that $q = 0$. This means that Wrongway always chooses to Wait.

- Subway doesn't actually have to fight Wrongway if they choose to Wait. How do we explain socially what is happening when Subway picks $p = 0.5$. What is the psychological reason for Wrongway choosing $q = 0$?
- Suppose that Subway has to choose to fight at the same time that Wrongway chooses whether to enter or not, so that an angry fighting Subway always spends some money even if Wrongway Waits. Then the payoff might be

	B1 (Enter)	B2 (Wait)	
A1 (C)	(1,2)	(2,1)	p
A2 (F)	(0,0)	(1,1)	1-p
	q	1-q	

What mixed strategies are there now?

- The same kind of mixed strategies can work with three or more options. Try it out with Rock-Paper-Scissors.

	B Rock	B Paper	B Scissors	
A Rock	(0,0)	(-1,1)	(1,-1)	p
A Paper	(1,-1)	(0,0)	(-1,1)	r
A Scissors	(-1,1)	(1,-1)	(0,0)	1 - p - r
	q	s	1 - q - s	

Now we have to solve the following algebraic problems:

$$1 - p - 2r = 2p + r - 1 = r - p$$

$$1 - q - 2s = 2q + s - 1 = s - q$$

What is the typical solution that we think of when we play Rock-Paper-Scissors? Are there others?

- Apply mixed strategies to the stag hunt and determine what proper mixed strategies the players should use.

Stag Hunt

	B1	B2
A1	$(2, 2)$	$(0, 1)$
A2	$(1, 0)$	$(1, 1)$

- Recall Sony and Toshiba (before they made decisions at different times). The game looked like this.

	B1 HD	B2 BR
A1 HD	$(3, 4)$	$(1, 1)$
A2 BR	$(2, 2)$	$(4, 3)$

If the players can play mixed strategies, what should they do?