UCI Middle School Math Circle: Game Theory

In the following problems each player wants to win. Determine who can always achieve this goal (i.e. who has the winning strategy) and why:

Symmetry

1) Two players take turns putting pennies on a round table, without piling one penny on the top of another. the player who cannot place a penny loses.

2) Two players take turns placing bishops on the squares of a chessboard, so that they cannot capture each other (the bishops may be placed on square of any color). The player who cannot move loses.

3) There are two piles of 7 stones each. At each turn, a player may take as many stones as he chooses, but only from one of the piles. The loser is the player who cannot move.

4) There are two piles of stones. One has 30 stones and the other has 20 stones. Players take turns removing as many stones as they please, but from one pile only. The player who cannot remove stone loses.

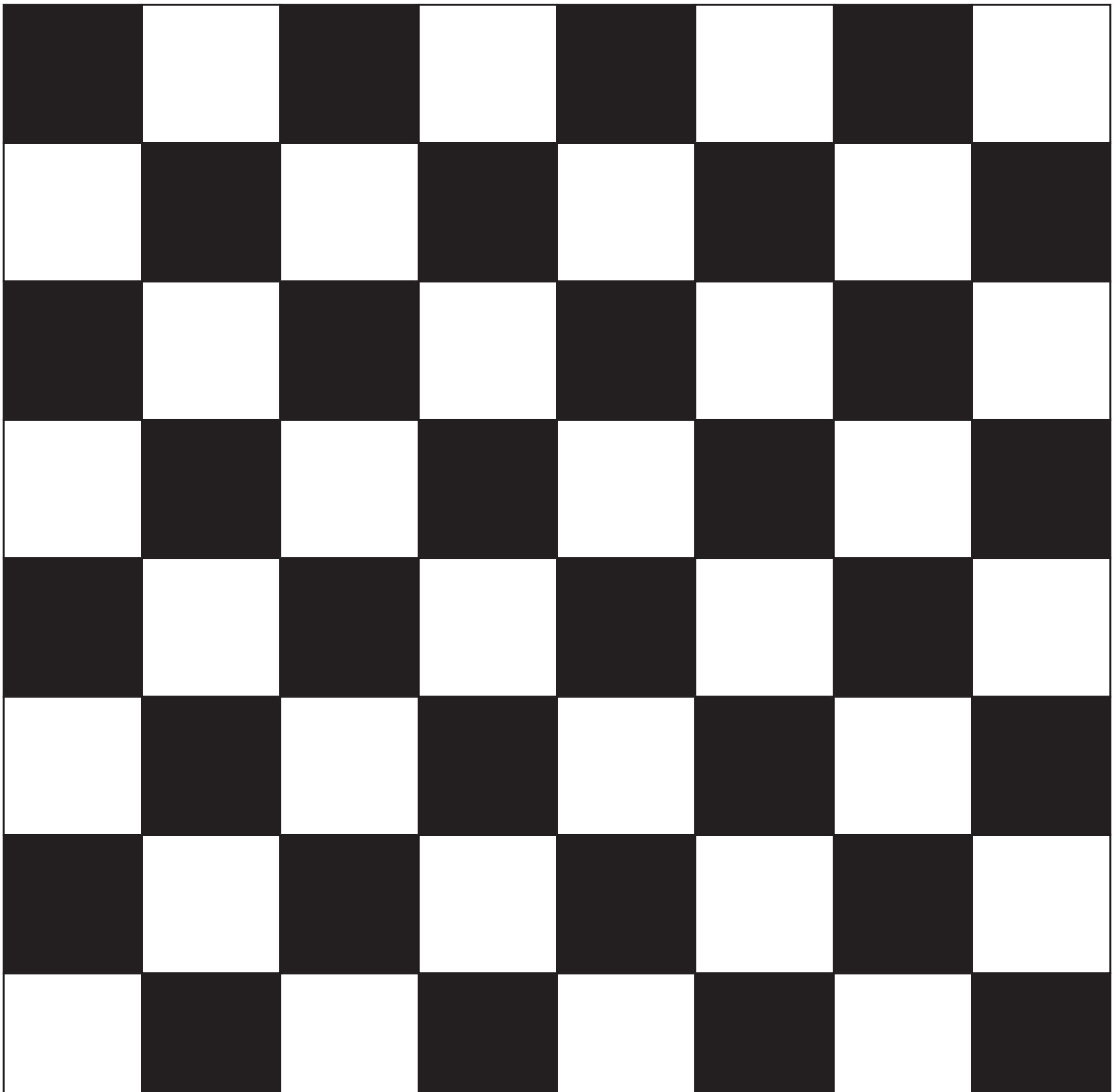
Winning Positions

1) On a chessboard, a rook stands on square a1. Players take turns moving the rook as many squares as they want, either horizontally to the right or vertically upward. The player who can place the rook on square h8 wins.

2) There are two piles of candy. One contains 20 pieces, and the other has 21. Players take turns eating all the candy in one pile, and separating the remaining candy in to two (not necessarily equal) non-empty piles. The player who cannot move loses.

3) A checker is placed at each end of a strip measuring 1x20. Players take turns moving either checker in the direction of the other, each time by one or two squares. A checker cannot jump over another checker. The player who cannot move loses.

4) A box contains 300 matches. Players take turns removing no more than half of the matches in the box. The player who cannot move loses.



UCI Middle School Math Circle: Game Theory bonus problems

1) This game begins with the number 2. In one turn, a player can add to the current number any number smaller than it. The player who reaches the number 1000 wins.

2) This game begins with the number 1000. In one turn, a player can subtract from the current number any power of two less than the number. The player who reaches the number 0 wins.

3) Two players take turns breaking a 6x8 chocolate bar along the lines which divide the chocolate squares. The first player who cannot divide the chocolate bar loses.

4) There are two piles of matches:
    a) a pile of 101 matches and a pile of 201 matches
    b) a pile of 100 matches and a pile of 201 matches

   Players take turns removing a number of matches from one pile which is equal to one of the divisors of the number of matches in the other pile. The player removing the last match wins.