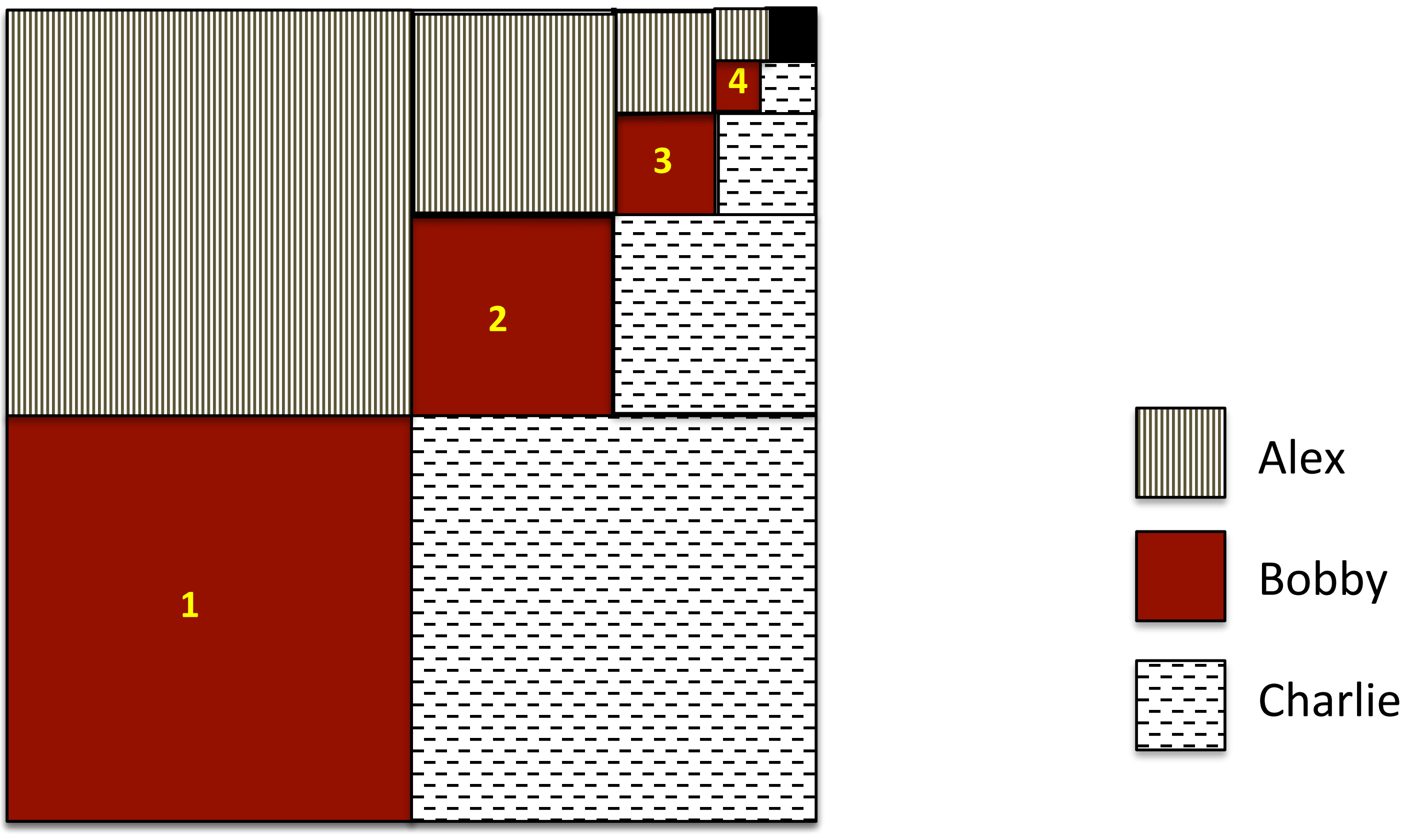
Chocolate Sums



Three twin brothers (Alex, Bobby and Charlie) share a square piece of chocolate. They split the chocolate in 4 parts, and each gets a piece. Then they keep splitting in 4 parts whatever is left, getting a piece each every time

1. Assuming that the entire chocolate is one unit high and one unit wide, so that the entire area is 1, how big are the pieces that Alex gets.

   Area of 1st piece: ______________ Area of 2nd piece: ______________

   Area of 3rd piece: ______________ Area of 4th piece: ______________

As Alex, Bobby and Charlie continue to split the chocolate bar, the pieces keep getting smaller and smaller. We will assume, however, that the brothers have a knife that can cut infinitely small pieces of chocolate, so the process go on forever.

2. Look at the area of the 5^{th} , 6^{th} , 7^{th} , …. pieces. Can you find the pattern? Hint: Factor the denominators of your fractions. Write a general expression for the area of the n^{th} piece that each brother gets.

   Area of n^{th} piece: ____________

3. Let's now look at how much chocolate is left…

   Area of left over chocolate after each brother gets his first 2 pieces: _______

   Area of left over chocolate after each brother gets his first 3 pieces: _______

   Area of left over chocolate after each brother gets his first 4 pieces: _______

4. What remains after the brothers have split the chocolate *n* times?

   Area of left over chocolate after each brother gets his first *n* pieces:

   ____

5. Finally, we look at the amount of chocolate the three brothers eat. Do all the brother eat the same amount of chocolate? Why or why not?

   ____

6. After the chocolate has been split 4 times, Alex ends up with the following amount of chocolate:

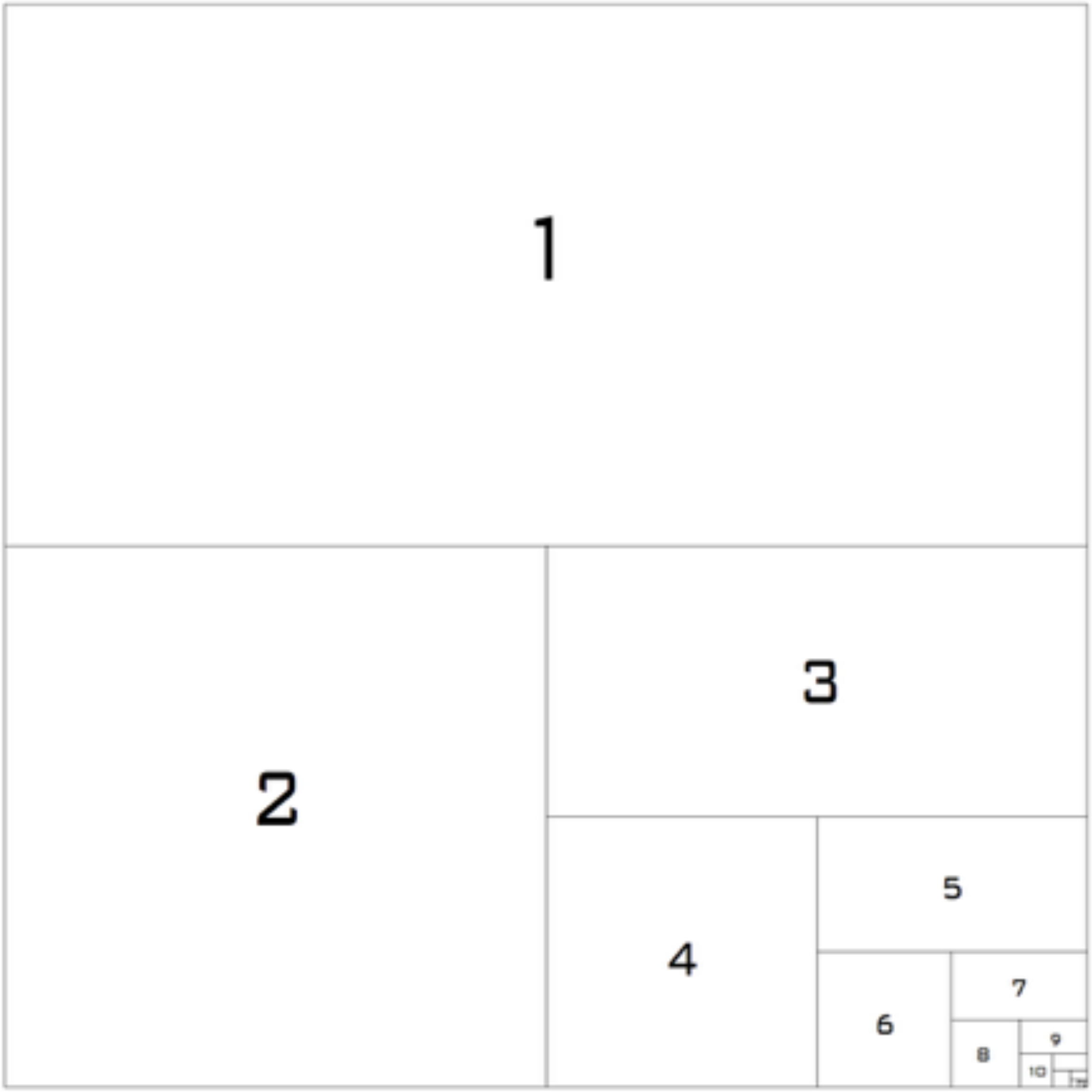
   (area of 1st piece) + (area of 2nd piece) + (area of 3rd piece) + (area of 4th piece) =

   ____

   Hint: Find a clever way to compute this sum. Think about the chocolate the brothers do not get to eat…

7. Assuming the brothers keep splitting the chocolate infinitely many times, what is the total area of chocolate Bobby will end up eating? *Hint: Refer to the picture.*

Little Boxes[1]



Imagine that the pattern shown above continues infinitely, so that the boxes in the lower right hand corner keep getting smaller and smaller. Also assume that the entire square is one unit high and one unit wide, so that its area is one; also assume that each (non-square) rectangle in the picture has short-side to long-side ratio 1/2.

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[1] Adapted from materials for the Julia Robinson Math Festival

1. What is the area of the rectangle labeled 1?

   What is the area of the square labeled 2?

   How about the rectangle labeled 3?

   The square labeled 4?...

   What is the pattern? Can you write down a general expression for the area of the rectangle labeled n?

2. What is the sum of the areas of rectangles 1 and 2?

   What is the sum of the areas of rectangles 1, 2 and 3?

   How about rectangles 1, 2, 3 and 4?

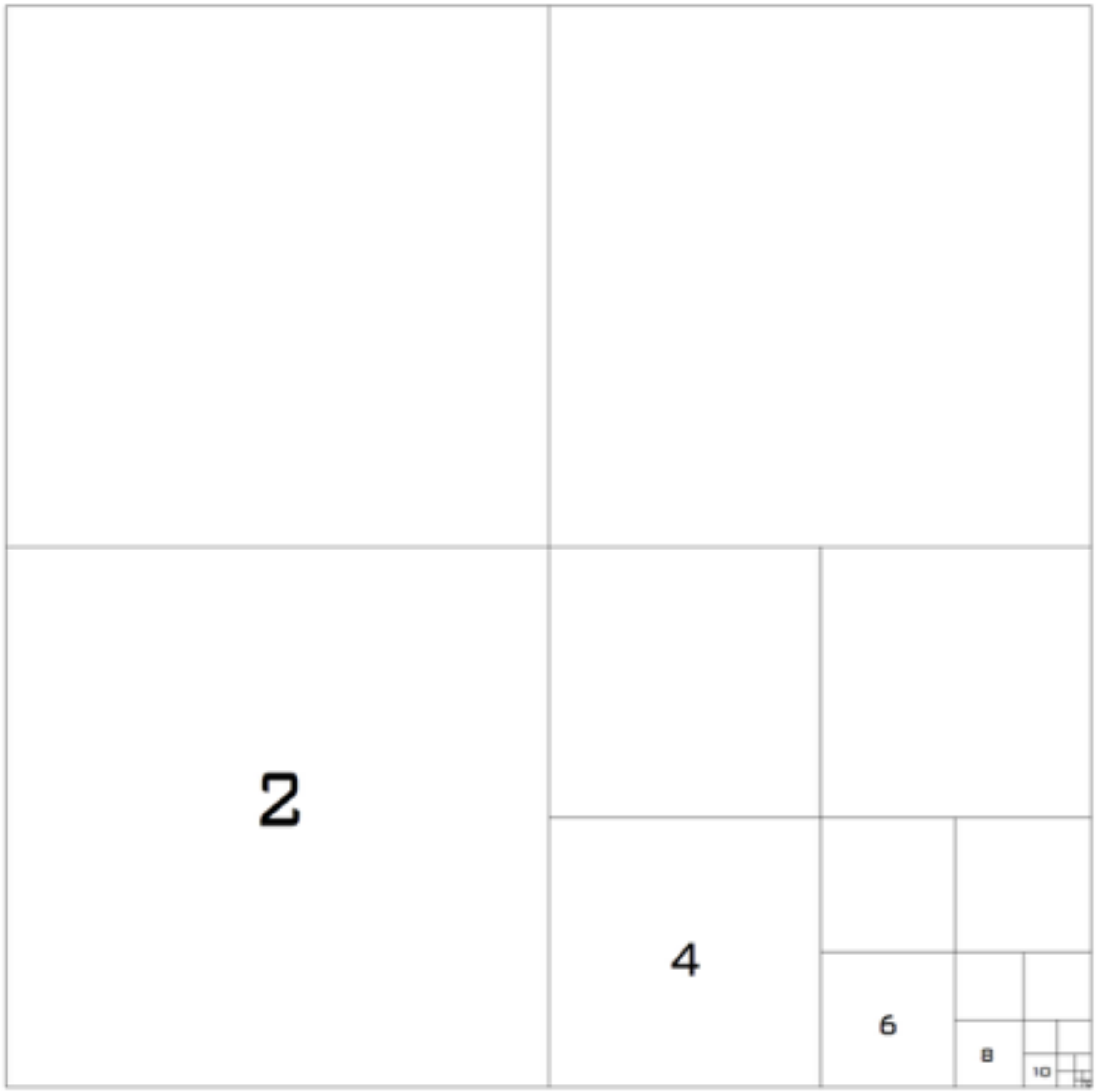
   Rectangles 1, 2, 3, 4 and 5?...

   What is the pattern here? Can you explain it in terms of the picture? Can you write down a general expression for the sum of areas 1 through n?

3. If we added up the area of all the rectangles, what would it be? Look at the picture for this one. Then think about your answer to problem 2.

   How many rectangles would you need to add to get more than 99% of the total area?

4. Wait a minute?!?!?! The sum of all the areas of the rectangles was a finite number? How can a sum of infinitely many numbers be finite?



Now let's think about just the even-numbered rectangles in the diagram (the squares).

5. What is the pattern of areas of the squares? Can you write a general expression for the area of the square labeled 2n?

6. What is the sum of areas of squares 2 and 4?

   What is the sum of the areas of squares 2, 4 and 6?

   How about squares 2, 4, 6 and 8?

   Squares 2, 4, 6, 8 and 10?

   What is the pattern here? Can you write down a general expression for the sum of the areas of squares 2 through 2n?

7. If we added up the area of all the squares, what would it be? There are two ways to answer this difficult question. One is to try to understand the pattern you found in problem 6, and where it's leading. The other is to think visually.

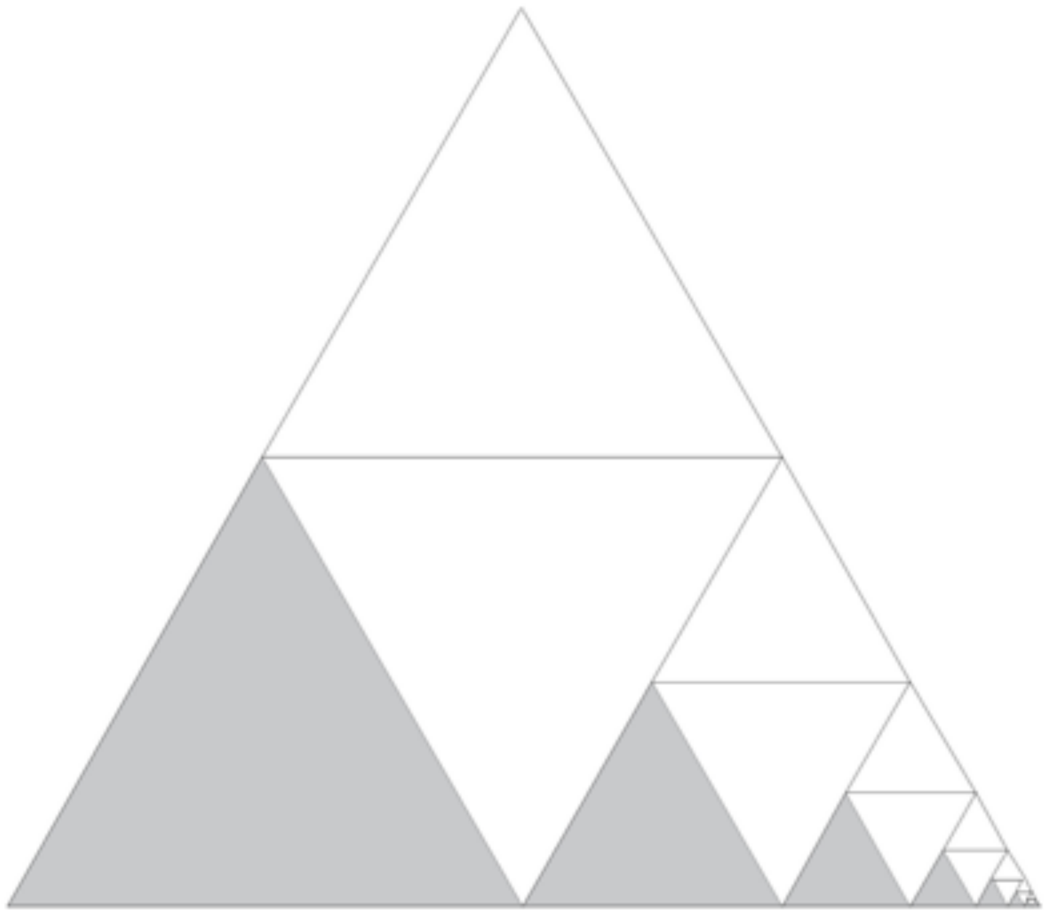
8. What is the sum of the areas of all the odd-numbered rectangles in the first diagram?

9. What is the sum of the areas of the rectangles labeled 4, 8, 12, 16, 20, etc. (every other square in the first diagram)?

10. What is the sum of the areas of the rectangles labeled 3, 6, 9, 12, 15, etc.

11. What is the sum of the areas of rectangles labeled with multiples of n, for any integer n?

12. Assuming that the area of the entire triangle in the diagram below is 1, what is the shaded area?



13. Can you draw pictures for other shapes? Can you divide the square or the triangle into pieces using other ratios?