

Game Theory

The UCI Math Circle

Penny Game

Now you and your partner play the penny game, each of you can choose either heads or tails. You can get one candy from your partner if your choices are the same. And if your choices are different, you need to give you partner one candy.

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

- (1) How many times did you play?
- (2) How many times did you win?
- (3) How many times did your partner win?
- (4) What would you choose if your partner chooses Heads?
- (5) What would you choose if your partner chooses Tails?

Guess The Average

Question:

What is median?

- (1) What is median of 0,1, 2, 3, 4, 8, 9, 20?
- (2) What is median of 0, 1, 2, 3, 4, 5, 100?

What is the average?

- (1) What is average of 0,1, 2, 3, 4, 8, 9, 20?
- (2) What is average of 0, 1, 2, 3, 4, 5, 100?

Now guess the average

Please write down a number you think you can win the game

- (1) Each of you and your classmates will select a number between 0 to 20. Then all the numbers will be collected. The winner will be the person who write down the number equal to the median.

- (2) Each of you and your classmates will select a number between 0 to 20. Then all the numbers will be collected. The winner will be the person who write down the number equal to $\frac{1}{2}$ of the median.

- (3) Each of you and your classmates will select a number between 0 to 20. Then all the numbers will be collected. The winner will be the person who write down the number equal to $\frac{1}{4}$ of the median.

Nash Equilibrium

Which box will you choose? You and your partner want to choose between two boxes A and B. After you read the following question, choose the one you think you will win more candies. Play this game with your partner 8 times for the first two games.

- (1) If both of you choose A, both of you will get two candies. If both of choose B, both of you will get one candy. If you and your partner choose different boxes, the person choose A will get four candies, the person choose B will get no candy.

	A	B
A	2, 2	4, 0
B	0, 4	1, 1

Now let your partner choose first, after knowing your partner's choice, which box will you choose to get more candies?

Now let your partner choose first again, this time you wouldn't see your partner's choice, which box will you choose?

Play the game again, at this time you and your partner choose the box at same time, which box will you choose?

After playing the game, please answer the following questions

- (1) What would you choose if your partner chooses A?
- (2) What would you choose if your partner chooses B?
- (3) What is your best strategy for this game?

(2) If both of you choose A, both of you will get one candy. If both of choose B, both of you will get two candies. If you choose box A and your partner choose B, you will get three candies, and your partner will get no candy. If you choose box B and your partner choose A, you will get no candy, your partner will get five candies.

	A	B
A	1, 1	3, 0
B	0, 3	2, 2

Repeat the same steps

Now let your partner choose first, after knowing your partner's choice, which box will you choose to get more candies?

Now let your partner choose first again, but this time you wouldn't see your partner's choice, which box will you choose?

Play the game again, at this time you and your partner choose the box at same time, which box will you choose?

After playing the game, please answer the following questions

- (1) What would you choose if your partner chooses A?
- (2) What would you choose if your partner chooses B?
- (3) What is your best strategy for this game?

(3) If both of you choose A, you will get four candies and your partner will get two candies. If both of choose B, you will get two candies and your partner will get four candies. If you and your partner choose different boxes, both of you will get one candy.

	A	B
A	4, 2	1, 1
B	1, 1	2, 4

Repeat the same steps

Now let your partner choose first, after knowing your partner's choice, which box will you choose to get more candies?

Now let your partner choose first again, but this time you wouldn't see your partner's choice, which box will you choose?

Play the game again, at this time you and your partner choose the box at same time, which box will you choose?

After playing the game, please answer the following questions

- (1) What would you choose if your partner chooses A?
- (2) What would you choose if your partner chooses B?
- (3) What is your best strategy for this game?

Now there are three boxes, A, B and C, you play the game again with the following table.

	A	B	C
A	4, 4	0, 0	2, 0
B	2, 2	3, 1	2, 1
C	2, 1	2, 2	1, 1

Please answer the following questions

- (1) What would you choose if your partner chooses A?
- (2) What would you choose if your partner chooses B?
- (3) What would you choose if your partner chooses B?
- (4) What is your best strategy for this game?

Subgame Perfect Nash Equilibrium

Suppose you and your partner play the game together. At the beginning, you need to decide whether you want to choose “start the game” or “stop the game”. If you choose not start, you will get two candies and your partner will get four candies. If you choose to start the game, then your partner will need to choose “continue the game” or “stop the game”. If your partner chooses stop, each of you will get one candy, and if your partner chooses to continue, you will get four candies, your partner will get two candies?

Please answer the following questions and think about what are the differences between this problem and the previous problems in the section of Nash Equilibrium.

- (1) Do you want to choose start the game?
- (2) Do you want your partner to choose continue or stop?
- (3) If you can know whether your partner will choose continue or stop before you choose start or not, do you want to choose start the game?
- (4) Do you think your partner want you to choose start the game or not?
- (5) What is your best strategy for this game?