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Level 1 Math Circle

Pigeonhole Principle

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Try to understand the following **Pigeonhole Principle**: Suppose you have k pigeonholes and n pigeons to be placed in them. If $n > k$ (number of pigeons > number of pigeonholes) then at least one pigeonhole contains at least two pigeons.

The following is called **Generalized Pigeonhole Principle**: If n pigeons are sitting in k pigeonholes, where $n > k$, then there is at least one pigeonhole with at least n/k pigeons.

Q1-1. Assume you have a mixture of black cards and red cards, what is the minimum number of cards needed before a pair of the same color can be guaranteed?

Q1-2. Assume you have a mixture of cards of 4 different suits, what is the minimum number of cards needed before a pair of the same suit can be guaranteed?

2. In the movie "Cheaper by the Dozen," there are 12 children in the family.

Q2-1. Prove that at least two of the children were born on the same day of the week.

Q2-2. Prove that at least two family members (including mother and father) are born in the same month.

Q2-3. Assuming there are 4 children's bedrooms in the house, show that there are at least 3 children sleeping in at least one of them.

Q3.Pigeonhole Elementary School has 500 students. Show that at least two of them were born on the same day of the year.

Q4.There are 50 baskets of apples. Each basket contains no more than 24 apples. Show that there are at least 3 baskets containing the same number of apples.

Q5. Choose 6 natural numbers (i.e. positive integers) from 1 to 10. Can you find two of the numbers you pick add up to 11? Try to do it a few times with different choices of numbers. Is it always possible to find two of the numbers you pick add up to 11? Try to explain it using the pigeonhole principle.

Q6. Find another person to play the following game. Each person takes turn writing down a natural numbers. Before your turn to write a number, you have choice to "catch" if you find the difference of two numbers that are written (regardless who writes it) is divisible by 7. The player who successfully catch is the winner. For example, player A write down 11, player B write down 19, player A write down 20, player B write down 25, player A catches because $25 - 11 = 14$ is a multiple of 7. In this game, what is the least amount of numbers that can guarantee a player can catch?

Q7. Show that for any natural number n there is a number composed of digits 5 and 0 only and divisible by n. (Hint: Use the conclusion of previous problem.)

Q8. Play the following game with a group of people (any amount). You can choose to shake or not shake hands with any player in this game. Write down the name of the people where you shake hands with. Write down how many people you shake hands with. In the end, can you always find 2 people shake hands with same amount of people? Can you explain this using the pigeonhole principle?

Q9 (Hard). In a group of 6 people, can you always find 3 mutual friends or strangers? Try to find a group of 6 people to check this fact. Can you explain this using the pigeonhole principle?

Q10. (Almost impossible). In a group of 9 people, can you always find 3 mutual friends or 4 mutual strangers? Try to find a group of 9 people to check this fact. Can you explain this using the pigeonhole principle?