PROGRAM

CBMS Conference: THE SOLUTION OF PROBLEMS IN MULTIPLY-CONNECTED DOMAINS

Principal Lecturer: Professor Darren Crowdy

Supported by:
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Organizing Committee:

- Bernard Deconinck (University of Washington)
- Stefan Llewellyn Smith (UC San Diego)
- Tom Trogdon (UC Irvine)

1 Schedule

All events will take place in Natural Sciences II, room 1201.

	M 6/18	Tu 6/19	W 6/20	Th 6/21	F 6/22
9:30	PL1	PL3	PL5	PL7	PL9
10:30	Break	Break	Break	Break	Break
11:00	GL1	GL2	PL6	GL3	GL4
12:00	Lunch	Lunch	–	Lunch	PL10
13:30	PL2	PL4	–	PL8	–
14:30	Break	Break	–	Break	DS4 (optional)
15:00 - 17:00	DS1	DS2	–	DS3	DS4 (optional)
17:00 - 20:00	–	PS	–	–	–

PL=Principal Lecturer, GL=Guest Lecturer (see below), DS=Discussion session, PS=Poster session & reception

2 Description of lectures

PL1 **The geometry of complex functions:** Introduction to hyperbolic geometry; Möbius maps and Schottky groups; algebraic curves and uniformization; multiply connected domains; categorization of canonical domains.

PL2 **The Schottky-Klein Prime Function:** What is the Schottky-Klein Prime Function? The Schottky double of multiply connected planar domains; the Prime Function of the Riemann sphere and the torus.

PL3 **Constructive potential theory:** First-type Green's functions in multiply connected domains; modified Green's functions in multiply-connected domains; harmonic measures; Poisson and Villat integral formulas for modified Schwarz problems and their higher genus extensions.

PL4 **Automorphic functions:** Elliptic functions as a paradigm; Weierstrass' σ, ζ and $\wp$ functions within the Schottky model; Abel's theorem; connection with theta functions.

PL5 **Multiply connected conformal mapping:** Conformal slit mappings as building blocks; construction of the Schwarz-Christoffel formula for holey domains; multiply connected polycircular-arc mappings.

PL6 **Cauchy transforms of planar domains:** Cauchy transforms of planar domains, relation to Biot-Savart law; balayage, the inverse moment problem; conformal geometric solution to inverse gravitational problem; connections to integrable systems (Toda/Whitham hierarchy).

PL7 **Quadrature domains and applications in physics:** Quadrature identities and quadrature domains; a denseness result; quadrature domains as a tool in fluid and solid mechanics.

PL8 **The geometry of Fourier transforms:** A geometric derivation of the Fourier transform and generalization; the Fokas transform for Laplace's equation in polygons.

PL9 **Fourier-Mellin transforms for circular domains:** Circular domains; Fourier-Mellin transforms for circular polygons; Fourier-Mellin transforms for multiply connected domains.

PL10 **Transform techniques for boundary value problems:** Mixed boundary value problems and applications; new transform methods for mixed BVPs.

3 Guest Lectures

GL1 Paul Wiegmann (`p-wiegmann@uchicago.edu`, University of Chicago)

Edge dynamics of a quantized vortex patch

A patch of a uniform vorticity of an inviscid incompressible fluid can move only by changing its shape. This is a well and long ago studied contour dynamics.

How does this dynamic change if the patch of vorticity consists of a large number discrete vortices, a vortex matter? The continuum limit of the vortex matter is the uniform vorticity. Does the discreetness of vortices yields a small correction to the contour dynamics? Apparently, effects of discreetness of vortices does not disappear in the continuum limit. A new mode of vorticity propagating along the edge of the patch emerges. This mode is governed by the celebrated Benjamin-Ono equation and features quantized solitons revealing quantized nature of vortices.

GL2 Tom Delillo (`delillo@math.wichita.edu`, Wichita State)

Some numerical methods for conformal mapping of multiply connected domains

We will give an overview of some examples of methods for approximating conformal maps to multiply connected domains. The methods use domains bounded by circles as the computational domains. The first class of methods is a generalization of Fornberg's method for simply connected domains and uses a Newton-like method to find the boundary correspondences and centers and radii of the circles. The map is represented as a sum of Laurent series centered at the circles. The second class of methods are Schwarz-Christoffel methods for multiply connected domains and a continuation method is used to find the prevertices and the centers and radii of the circles. The relation of these methods to linear Riemann-Hilbert problems for circle domains, the Schottky-Klein prime function, ill-conditioning due to crowding, smoothing of corners by powers maps, and some selected applications will also be discussed, time permitting.

GL3 Anna Zemlyanova (`azem@ksu.edu`, Kansas State)

Complex analysis methods in fracture problems with surface elasticity

The talk will focus on modeling of fracture with surface elasticity in the Steigmann-Ogden form. Introduction of surface elasticity on the crack boundaries leads to non-classical boundary conditions on the banks of the fracture. The boundary conditions combine tractions on the crack bound- aries and derivatives of the displacements of these boundaries. It will be shown that the mechanical problem can be reduced to systems of singular integro-differential equations using methods of complex analysis. The sys- tems of singular integro-differential equations will be be further regularized and the existence and uniqueness of the solution for almost all the values of the parameters will be proved. A problem of a straight mode I/II non- interface fracture in an infinite plate will be considered in detail. It will be shown that modeling of fracture with the Steigmann-Ogden elasticity produces the stress and strain fields which are bounded at the crack tips. Moreover, it will shown that introduction of the surface mechanics into the modeling of fracture leads to size-dependent equations. A numerical scheme of the solution of the systems of singular integro-differential equations will be presented, and the numerical results will be given for different values of the mechanical and the geometric parameters.

GL4 Tom Trogdon (ttrogdon@math.uci.edu, UC Irvine)

A computational theory for Riemann–Hilbert problems

Riemann–Hilbert (RH) problems are boundary value problems posed on the complex plane. In this talk, I will give an overview of a theory for understanding the development and the convergence of numerical methods for RH problems. This includes using a notion of smoothness for functions defined on contours with self-intersections and building the associated Sobolev spaces. Applications to integrable systems and Weiner-Hopf problems will be given, including a discussion of the development of numerical methods that are asymptotically accurate.

4 Poster Session: Tuesday, June 19 at 5pm

1. Peter Baddoo (baddoo@damtp.cam.ac.uk, University of Cambridge)

   Vortex Equilibria in Ground Effect

   The vortex equilibrium configurations for an inclined flat plate close to the ground, in uniform flow, are investigated. By using the Schottky-Klein prime function to establish a conformal map from a canonical annular domain, analytical expressions for the complex velocity field are calculated for N vortices. Focusing on the single vortex case, the equilibrium positions, and corresponding intensity and plate circulation are calculated using the method of Brownian Ratchets. This method yields three loci of solutions with satisfy the Kutta condition of smooth flow at the trailing edge. As well as the classical leading and trailing edge loci, a third locus of vortices that are not attached to the plate is observed. The stability properties of these configurations are investigated and the vortex shedding from the trailing edge is modelled using the Brown-Michael equation. By solving a Riemann-Hilbert problem on the flat plate, the Schwarz problem for a pitching foil is solved, with implications for an oscillating, heaving foil in ground effect.

2. Anthony Davis (amdavis@ucsd.edu, UC San Diego)

Pressure-driven Flow through an L-Shaped Channel

Two contrasting solutions are presented. One uses Meleshkos complimentary sums technique for lid-driven cavity flow. The other uses Fokas unified transform method to solve for each of two Laplacian functions in a right-angled 45° triangular region.

3. Tom Delillo (delillo@math.wichita.edu, Wichita State)

   Applications of Numerical Conformal Mapping

   This poster summarizes thesis works by three graduate students at Wichita State University. Numerical conformal mapping techniques are applied to Stokes waves computations (J. Mears), multiply connected Riemann-Hilbert problems (R. Balu), and potential flow past multi-element airfoils (S. Sahraei).

4. Jordan Christian Hauge (jch14@ic.ac.uk, Imperial College London)

   A new method for Diffusion-Wavefield theory

   We demonstrate how the unified transform method developed by Fokas and collaborators can be extended to provide new solution methods for a class of boundary value problems arising in an area known as diffusion-wave field theory. [Joint work with Darren G. Crowdy].

5. Tae Eun Kim (kim.3562@osu.edu, Ohio State)

   Quasi-Solution Approach to Translating Pair of 2-D Vortex Patches

   We investigate the steady Euler equation for a symmetric pair of two-dimensional translating vortex patches. The problem is formulated in terms of a univalent map between the annular region $\{\zeta : \rho < |\zeta| < 1\}$ and the exterior of patch in the upper-half z-plane, whose real axis serves as the axis of symmetry. Using the method of quasi-solutions, a numerically obtained approximate analytical solution is used to reduce the strongly nonlinear problem into a weakly nonlinear analysis; the contraction mapping theorem over a suitably chosen solution space provides the existence and the uniqueness of solution for different values of $\rho \in (0, 1)$, where values closer to 1 correspond to more deformed patch shapes. Rigorous error bounds are also provided.

6. Vikas Krishnamurthy (vikas.krishnamurthy2@gmail.com, Erwin Schrodinger International Institute)

   Computing the Schottky-Klein prime function in asymmetric multiply-connected domains

   We present an algorithm for computing the Schottky-Klein prime function for a multiply-connected domain without symmetry. When the multiply connected domain is not symmetric with respect to the unit circle, conventional arguments for computing the prime function through the use of integrals of the first kind and Laurent series methods have to be revised. We will describe how this can be done and showcase its numerical implementation for selected asymmetric domains inspired by physical applications. Extensions to arbitrary multiply connected domains in the complex plane will be discussed.

7. Elena Louca (elouca@eng.ucsd.edu, UC San Diego)

Numerical solution of Wiener-Hopf problems using a Riemann-Hilbert formulation

A fast and accurate numerical method for the solution of scalar and matrix Wiener-Hopf problems is presented. The Wiener-Hopf problems are formulated as Riemann-Hilbert problems on the real line, and the numerical approach of such problems (see e.g. Trogdon & Olver (*SIAM*, 2015) is employed. It is shown that the known far-field behaviour of the solutions can be exploited to construct tailor-made numerical schemes providing accurate results. A number of scalar and matrix Wiener-Hopf problems that generalize the classical Sommerfeld problem of diffraction of plane waves by a semi-infinite plane are solved using the new approach. Joint work with Stefan G. Llewellyn Smith.

8. Hiroyuki Miyoshi (`hiroyuki_miyoshi@ipc.i.u-tokyo.ac.jp`, University of Tokyo)

   Binary CT reconstruction based on quadrature domains

   An algorithm for computed tomography(CT) from the small number of projections is proposed. When the density function is binary, the problem is formulated as localization for quadrature nodes in the multiply connected domain.

9. Liam Morrow (`liam.morrow@hdr.qut.edu.au`, Queensland University of Technology)

   Numerical solutions of multiply connected fluid flow problems

   One of the most well-studied problems in fluid dynamics is Hele-Shaw flow, which describes the motion of two immiscible fluids between two plates separated by a narrow gap. The problem has received significant attention largely due to the interfacial patterns which form due to the Saffman-Taylor instability. Mathematically, this is modelled with a non-linear moving boundary problem, where one fluid is surrounded by an infinite body of another fluid. Here we consider the evolution of a fluid surrounded by a finite amount of another fluid which results in two, or more, interfaces to be solved for. We present a robust level set based numerical scheme, and show that it is capable of reproducing the characteristic Saffman-Taylor instabilities, as well as describe the interaction between the two interfaces.

10. Béla Nagy (`nbela@math.u-szeged.hu`, HAS and University of Szeged)

    Some results about polynomials on general sets of the complex plane

    The purpose is to show some earlier results and a currently active project about the behavior of polynomials on a general, not necessarily simply connected domains. The results are sharp and heavily use potential theory and complex functions. The first topic is a sharpened form of Hilberts lemniscate theorem for a general class of sets and its applications: a Bernstein type inequality and an asymptotics for the Christoffel functions. The second topic is again motivated by search for sharp inequality for rational function on domains. This leads to the third topic which is an ongoing project about opening up set of Jordan arcs.

11. Jim Thomas (`jimthomas.edu@gmail.com`, WHOI and Dalhousie University)

    Amplitude equations for ocean waves

    This poster will discuss the application of multi-time-scale asymptotic methods for the derivation of amplitude equations for ocean waves. Three types of waves will be considered: internal gravity waves, surface gravity waves, and acoustic waves. The amplitude equations faithfully capture complex and intricate features of the wavefield, while being much faster to numerically integrate. Numerical simulations will be presented to argue that in suitable parameter regimes, the amplitude equations can replace the full set of nonlinear governing equations.

12. Jeremy Upsal (`jupsal@uw.edu`, University of Washington)

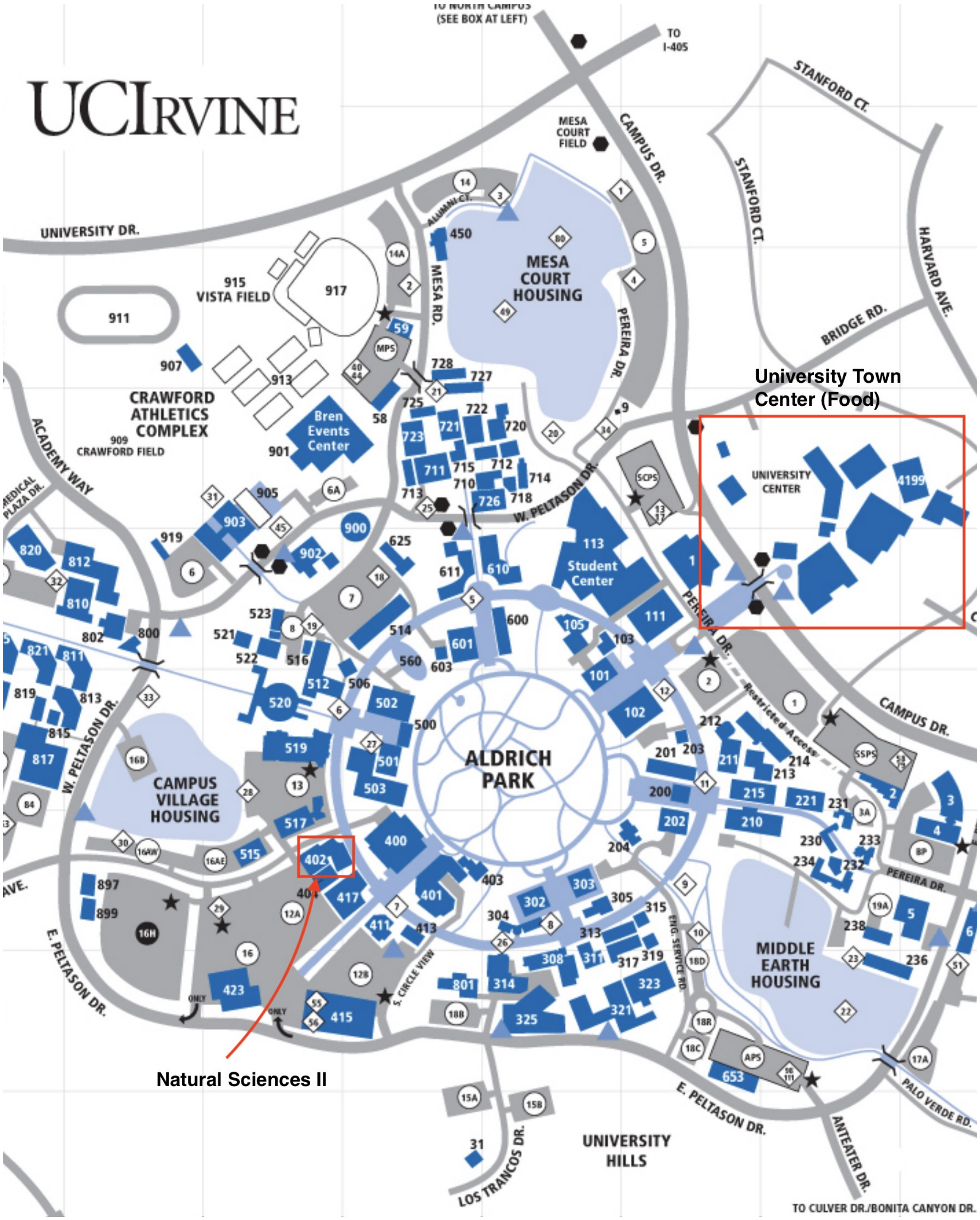
    On the orbital stability of elliptic solutions to focusing NLS

    The focusing Nonlinear Schrodinger Equation admits a class of standing wave solutions, expressed in terms of elliptic functions. The linear stability of these solutions with respect to subharmonic perturbations (i.e., perturbations whose period is a multiple of the solution period) is well understood using techniques which rely on integrability. In this talk I show that the linear stability results can be strengthened to orbital (nonlinear) stability by using the conserved quantities of NLS to construct a Lyapunov functional.

13. Xin Yang (`yangxin@uw.edu`, University of Washington)

    Numerical inverse scattering for the sine-Gordon equation

    We implement the numerical inverse scattering transform (NIST) for the sine-Gordon equation in laboratory coordinates on the whole real line using the method developed by Trogdon, Olver and Deconinck. The NIST allows one to compute the solution at any x and t without having spatial discretization or time-stepping. The numerical implementation is fully spectrally accurate. With the help of the method of nonlinear steepest descent, the NIST is demonstrated to be asymptotically accurate for large x and t.


UCIRVINE
TO NORTH CAMPUS (SEE BOX AT LEFT)
TO I-405
UNIVERSITY DR.
MESA COURT HOUSING
CRAWFORD ATHLETICS COMPLEX
909 CRAWFORD FIELD
915 VISTA FIELD
Bren Events Center
ACADEMY WAY
W. PELTASON DR.
113 Student Center
UNIVERSITY CENTER
University Town Center (Food)
ALDRICH PARK
CAMPUS VILLAGE HOUSING
MIDDLE EARTH HOUSING
UNIVERSITY HILLS
CAMPUS DR.
PEREIRA DR.
STANFORD CT.
HARVARD AVE.
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TO CULVER DR./BONITA CANYON DR.
Natural Sciences II