



# THE PLAN

An overview of what we need to do

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# A SUGGESTION

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Do we want to set aside (part of)  
Friday afternoon for a problem session?

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# AN ASIDE

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The book we discussed yesterday about the early history of descriptive set theory is:

**Naming Infinity: A True Story of Religious  
Mysticism and Mathematical Creativity**

**Lorne Graham and Jean-Michel Kantor**

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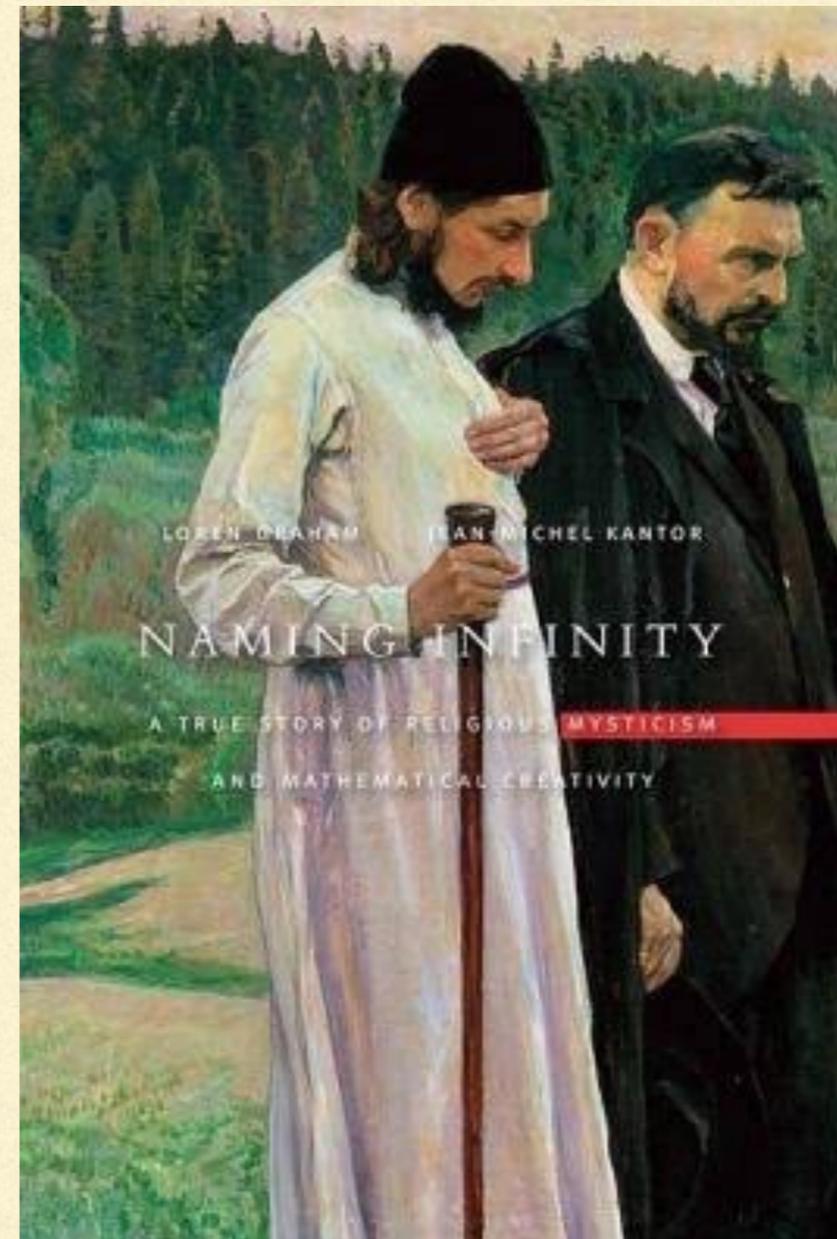
# AN ASIDE

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# OUR GOAL

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Show that isomorphism for ergodic diffeomorphisms is not a Borel equivalence relation.

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# OUR GOAL

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To do this we take

- The simplest possible space ( $Z$ )
  - The simplest possible manifold (a compact surface).
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# WHAT DID WE KNOW?

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Annals of Mathematics **173** (2011), 1529–1586  
doi: 10.4007/annals.2011.173.3.7

## **The conjugacy problem in ergodic theory**

By MATTHEW FOREMAN, DANIEL J. RUDOLPH, and BENJAMIN WEISS

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## Problem:

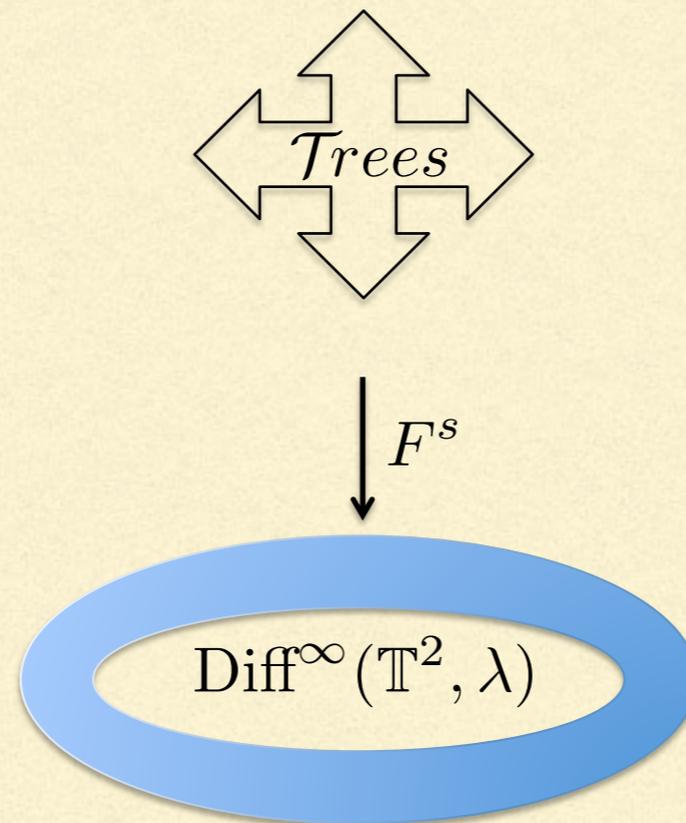
The reduction of ill-founded trees to isomorphism had its range in the transformations with odometer factors.

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# WE BUILD A REDUCTION

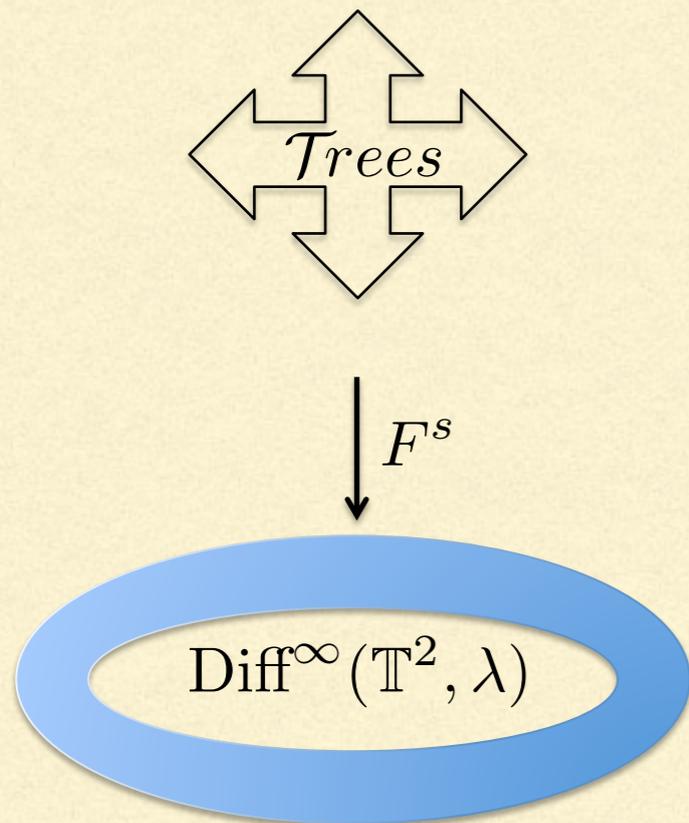
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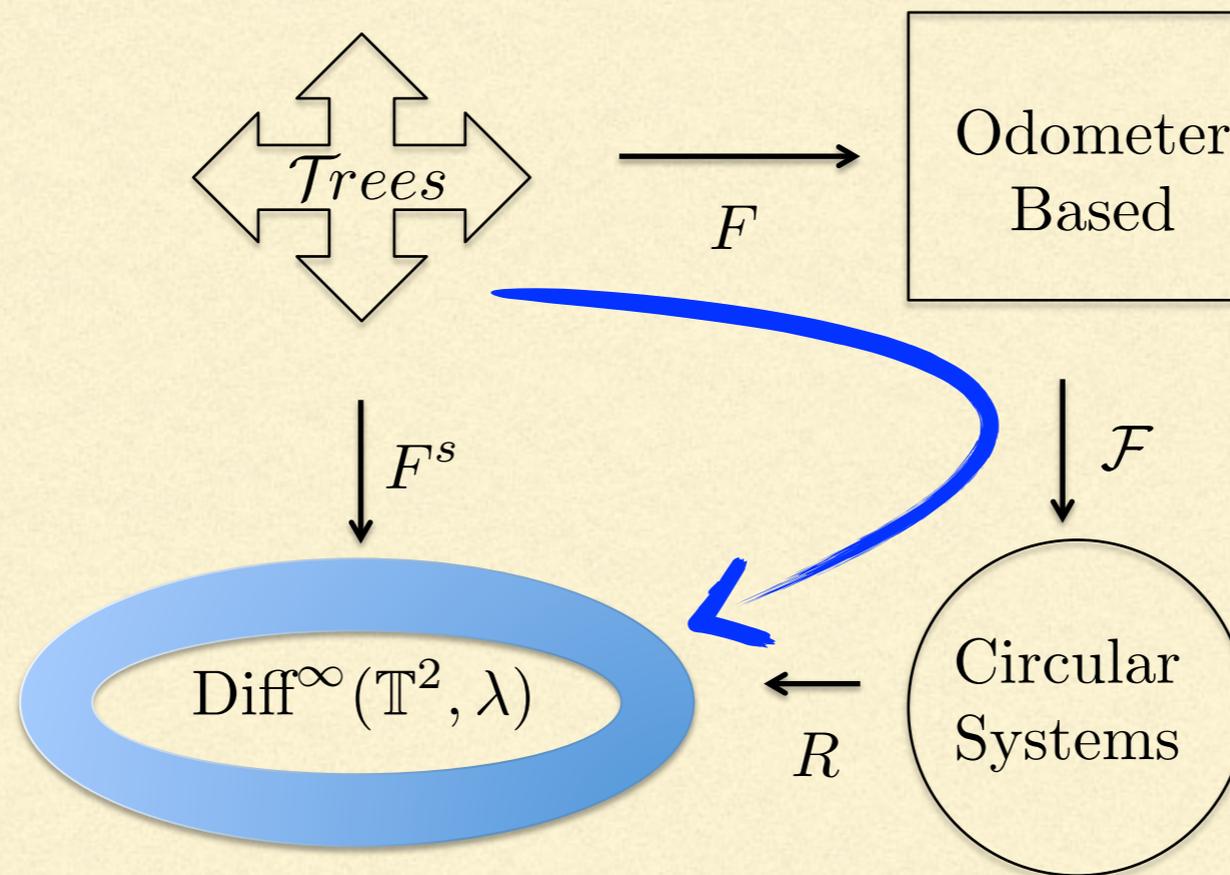
# WE BUILD A REDUCTION

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- $\mathcal{T} \mapsto_{F^s} (S, T)$
  - $S, T$  are ergodic.
  - $\mathcal{T}$  is ill-founded if and only if  $S \cong T$ .
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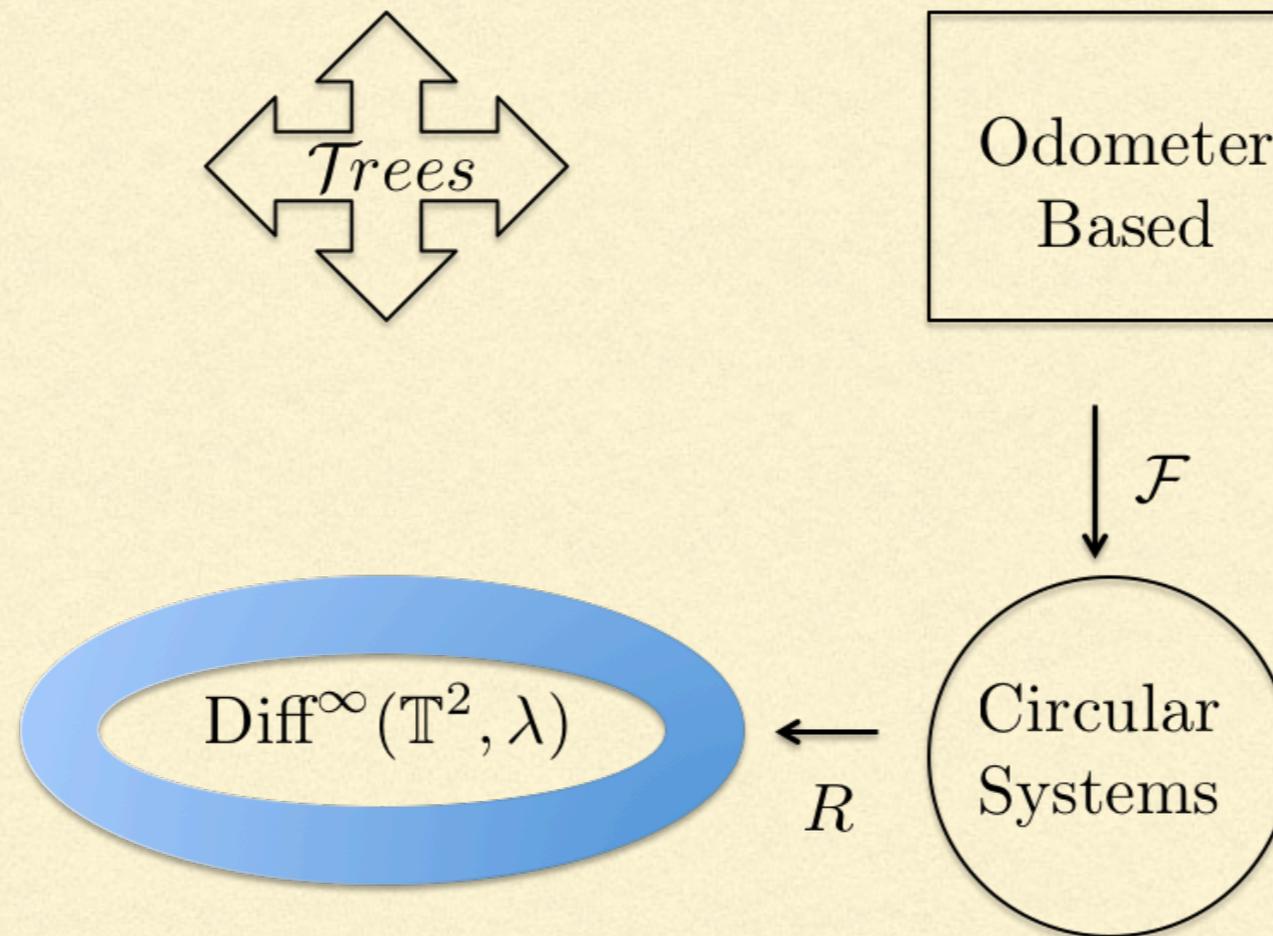
# INTERMEDIATE STEPS



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# WE GOT LUCKY

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Explain each step with lots of  
evocative **false facts** ...

*Illegal thinking!*

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# FIRST STEP

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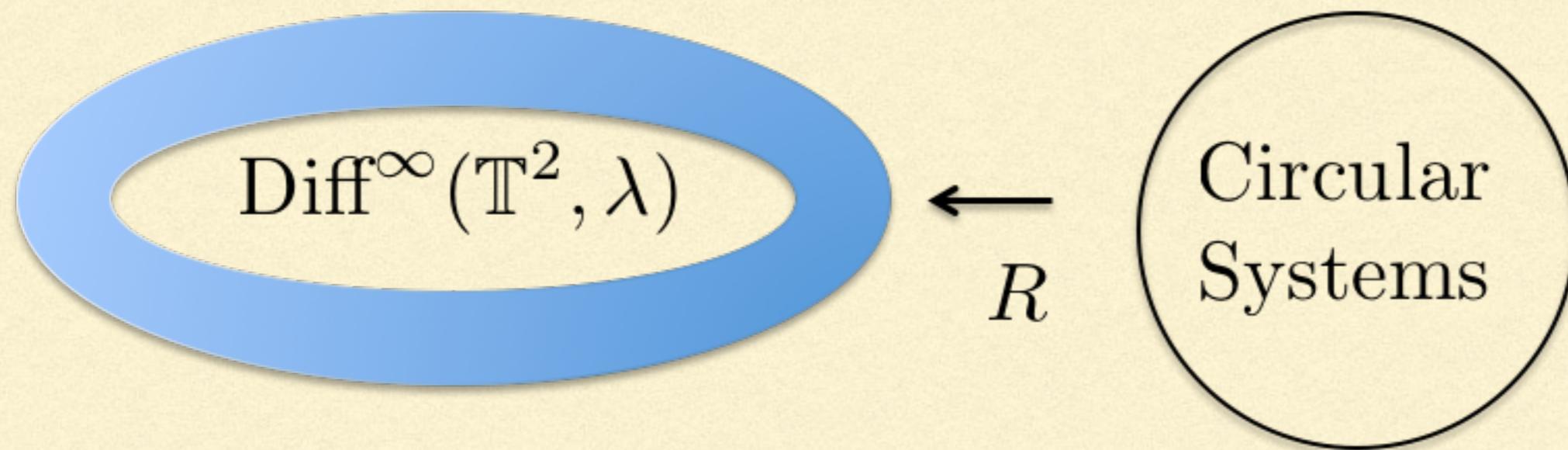
Remove geometry in favor of combinatorics/counting

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# APPROXIMATION BY CONJUGACY

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# SECOND STEP

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Build two categories and show they are isomorphic.

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# SECOND STEP

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Build two categories and show they are isomorphic.

- One category will be the odometer based systems
  - The other will be the circular systems
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CATEGORY ISOMORPHISM IS  
DIFFERENT THAN REDUCTION

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# CATEGORY ISOMORPHISM DIFFERENT THAN REDUCTION: STRONGER

As a consequence we can deduce that  
lots of structure for abstract MPT's can  
be realized by measure preserving  
diffeomorphisms of manifolds

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CATEGORY ISOMORPHISM IS DIFFERENT  
THAN REDUCTION: WEAKER

The functor is not defined on all factor  
maps—so not a reduction of  
isomorphism in general.

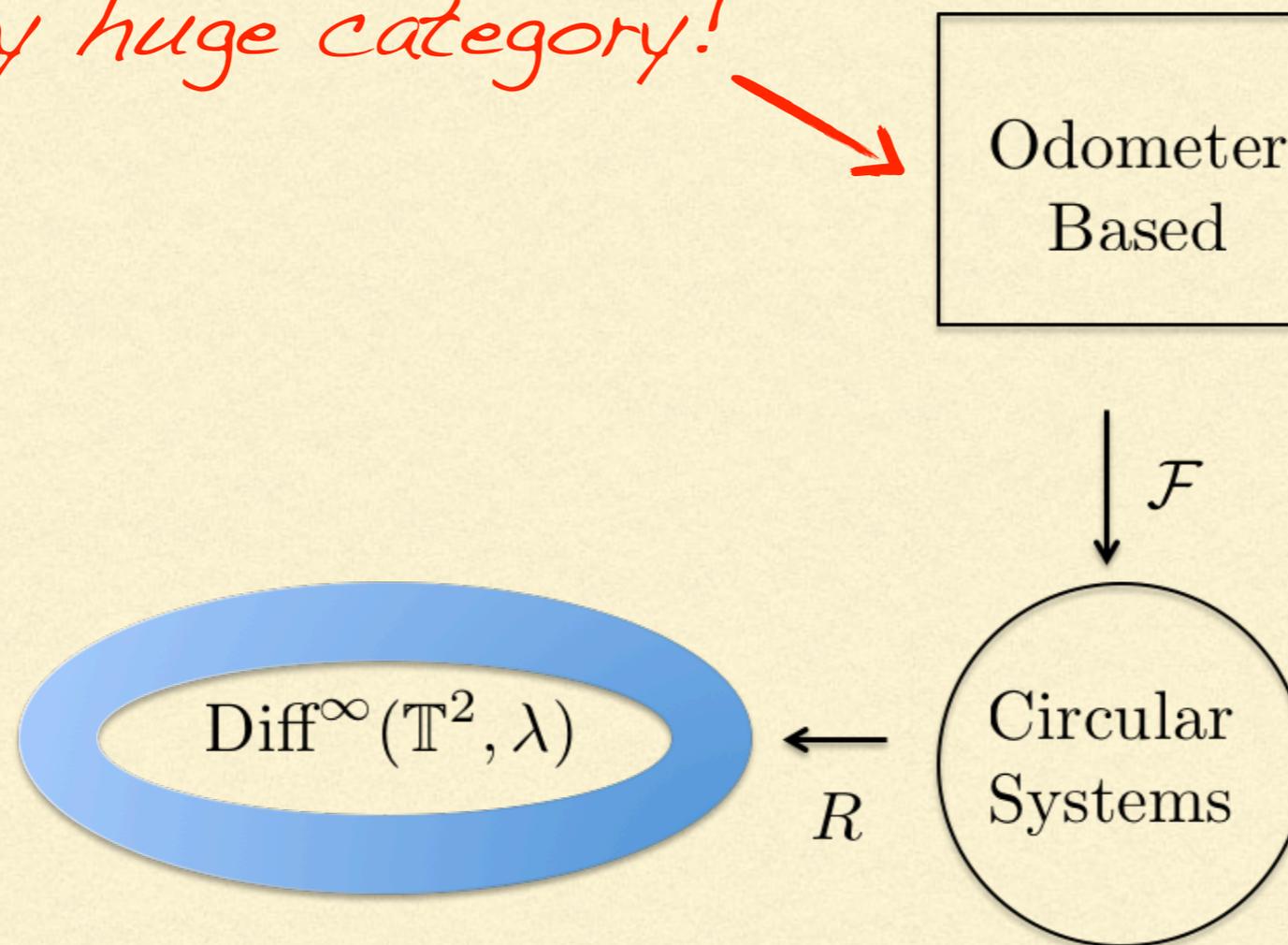
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# THE PICTURE

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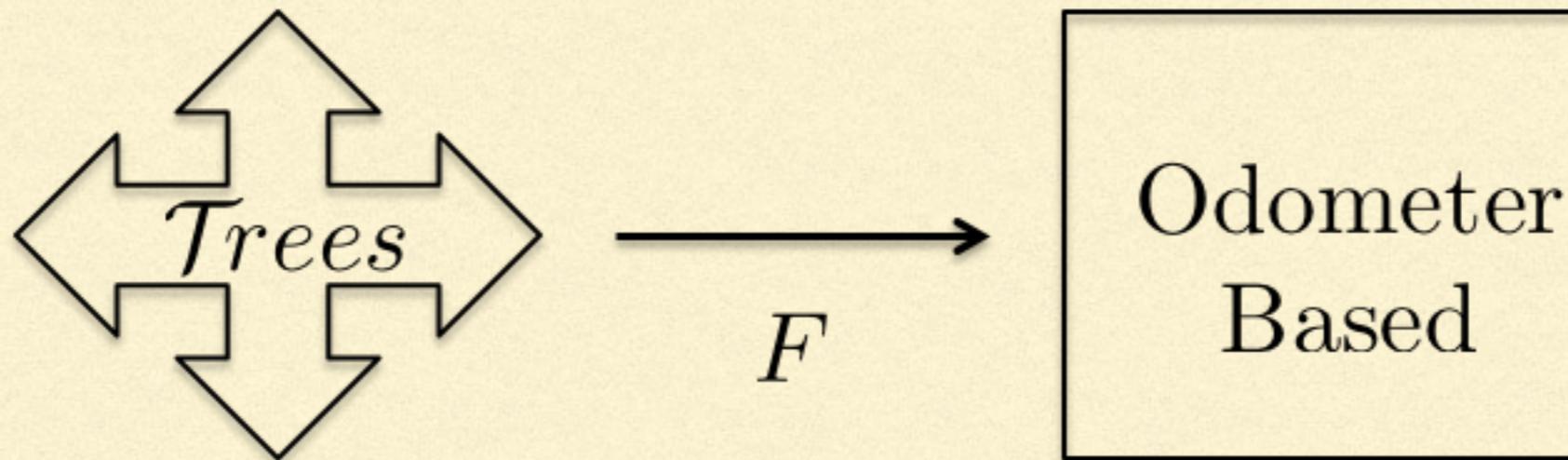
*A truly huge category!*



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# THE LAST PIECE

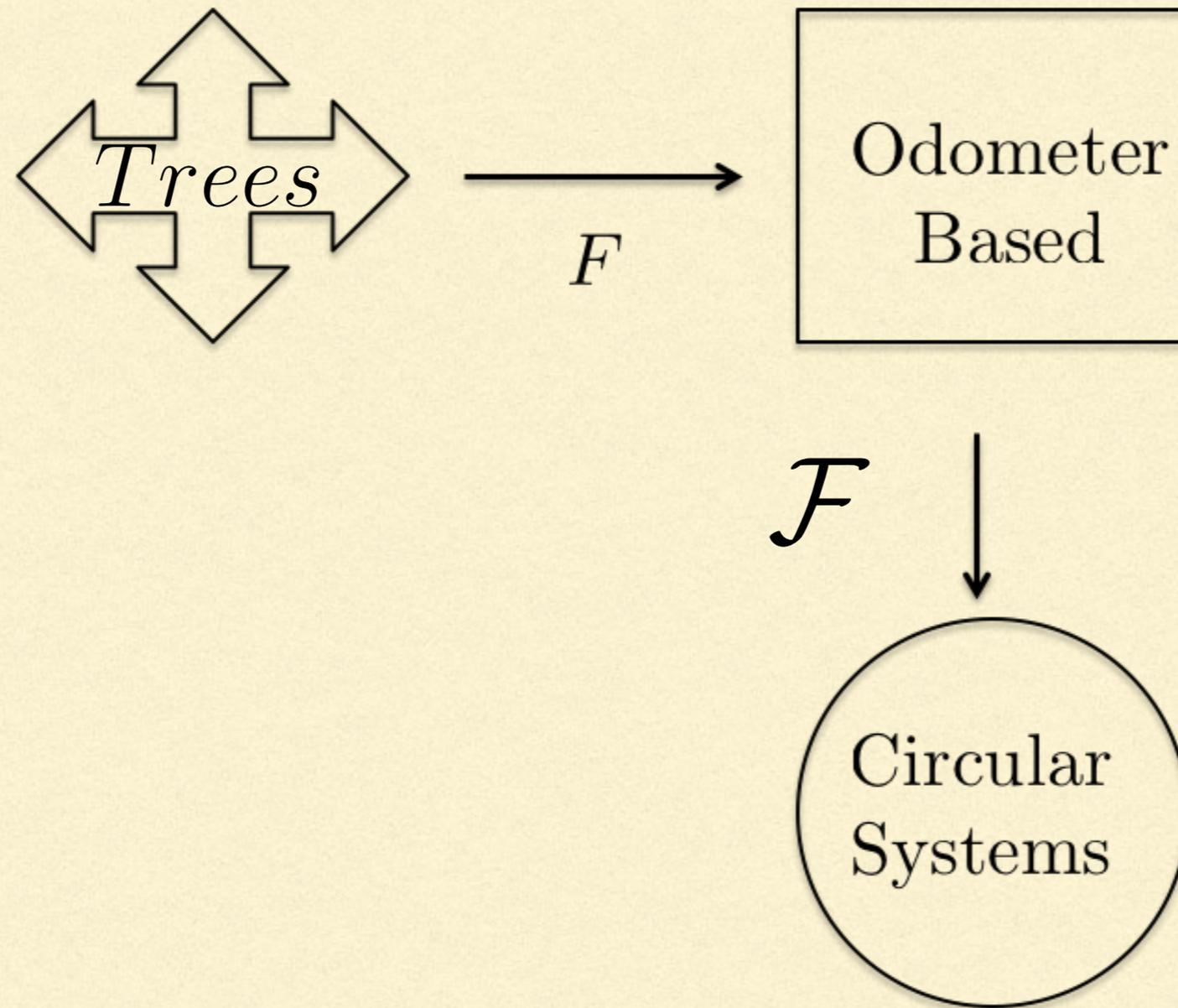
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WE NEED TO COMPOSE WITH  $\mathcal{F}$

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# THE PLAN

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- Discuss realization by diffeomorphisms
  - Build the functor
    - nuanced understanding of the ergodic theorem for symbolic systems
    - argue the functor preserves distal height, invariant measures, slow entropy (etc. etc.)
  - Build the reduction to odometer based systems that composes well with the functor.
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