Questions from UCI
Set Theory Summer School '17

September 26, 2017

1. (Gaebler) Is the isomorphism problem for ergodic measure preserving transformations on an infinite measure space complete analytic?

2. (Gaebler) Is there a reasonable notion of Bernoulli shift for infinite measure spaces?

3. (Thornton) Can Downarowicz's theorem [about Toeplitz sequences] be strengthened to a reduction of affine homeomorphism on a generic class of simplexes to topological isomorphism? (Note: affine homeomorphism is universal among orbit equivalence relations of Polish group actions, so cannot be reduced to isomorphism for Toeplitz shifts.)

4. (Monzavi) Suppose Γ is sofic and acts by measure preserving transformations on a standard probability space (X, μ). If Σ is a diffuse sofic approximation to Γ, does it follow that $h_\Gamma^\Sigma(X, \mu) \neq -\infty$. (This is known for trivial actions, and there may be known counterexamples.)

5. (Panagiotopoulos) Under what conditions does a class of measure preserving transformations admit a projective Fraïssé limit?

6. (Seward) Given a residually finite group Γ, for any decreasing sequence of subgroups $\langle G_n : n \in \omega \rangle$ such that $[G : G_n] < \infty$ and $\bigcap_{n \in \omega} G_n = \{e\}$, and X an algebraic shift of finite type we have

$$h_G^{(G_n)}(X, \text{ Haar measure}) = \limsup_{n \to \infty} \frac{1}{[G : G_n]} \log |\{x \in X : G_n \subseteq \operatorname{stab}(x)\}|.$$

   Excluding sequences $\langle G_n : n \in \omega \rangle$ where the set above is eventually empty, does this number actually depend on the sequence of subgroups?

7. (Lin) Is the F invariant of a Markov shift always the logarithm of an algebraic number? Or of a computable number?

8. (Thornton) Given a group Γ, consider continuous first order language of measure algebras equipped with a function symbol for each element of Γ. In this language, we can axiomatize the class of free Γ actions on measure

algebras. Does this class admit an existential closure? What is its type space? What are its stable models?

9. (attributed to Bowen) Is the collection of probability measure preserving F_2-actions isomorphic to a Bernoulli shift a Borel set?

10. (Foreman) Is there a reduction of a turbulent equivalence relation to the relation of isomorphism of ergodic Lebesgue measure preserving diffeomorphisms? Related to this: is there a generic class G of (abstract) measure preserving transformations and an odometer $\mathcal{O}$ such that the map $T \mapsto T \times \mathcal{O}$ preserves *non*-isomorphism on G.

11. (Foreman, Rudolph, Weiss) The isomorphism relation on rank-one measure preserving transformations is Borel, but cannot be reduced to an S^∞-action. Is there a structure theorem for the rank-one transformations? A weaker question is what the Borel complexity of the isomorphism–relation–on–rank–one is.

12. (Foreman) Is the *topological* isomorphism problem for diffeomorphisms complete analytic?

13. (Discussion during/after lecture) Can you characterize circular systems explicitly in ergodic-theoretic terms? More precisey is the set of transformations isomorphic to a circular system a Borel set.

14. (Discussion during/after lecture) Circular systems are defined in terms of sequences $\langle k_n, l_n : n \in \mathbb{N}\rangle$. How much do they depend on the sequences rather than the limiting irrational $\alpha = \lim \alpha_n$? In particular, does the collection of central values depend only on α?

15. (Thouvenot?) Can an arbitrary irrational rotation be realized as a diffeomorphism of the torus/disk? Hermann's theorem refers to rotation numbers, and doesn't directly forbid this.