Basic introductory books on ergodic theory are [31, 48]. More advanced books on ergodic theory are [28, 18].

The Rokhlin Lemma is likely proven in each of the books [31, 18, 28, 48].

The Borel Ergodic Decomposition Theorem is proven independently in [14, 47]. I believe shorter proofs may exist for the weaker result stating that every fixed invariant measure admits an ergodic decomposition, but I don’t have a reference for this.

If I remember correctly, a proof of the Becker–Kechris theorem can be found in [17]. Also, the book [17] is a good general reference for dynamics from a descriptive set theoretic point of view.

The theorem that every aperiodic Borel action of a countable group admits a countable generating partition is proven in [22] (relying upon [12] which proved this theorem for actions of Z). For actions of Z I believe this was independently proven by Rokhlin and/or Weiss, but I don’t remember a reference for this.

Actions of Z having discrete spectrum are classified in [18], with proofs included.


Ornstein and other researchers continued to cultivate the techniques used in the isomorphism theorem, ultimately creating ‘Ornstein theory,’ a collection of practical necessary-and-sufficient conditions for Z-actions to be isomorphic to Bernoulli. This led to the surprising discovery that many natural examples of Z-actions turn out to be isomorphic to Bernoulli, such as: factors of Bernoulli shifts [35], inverse limits of Bernoulli shifts [36], ergodic automorphisms of compact metrizable groups [30, 32], mixing Markov chains [15], geodesic flows on surfaces of negative curvature [37], Anosov flows with a smooth measure [40], and two-dimensional billiards with a convex scatterer [19]. These discoveries are generally considered to be the deepest results in entropy theory.

Entropy was extended from actions of Z to actions of amenable groups by Kifer in 1975 [29]. The main reference for the entropy theory of actions of amenable groups is [38], which extends Sinai’s factor theorem and Ornstein’s isomorphism theorem to amenable groups. This paper also included the “Ornstein–Weiss example” of the Bernoulli 2-shift factoring onto the Bernoulli 4-shift for the rank 2 free group.

In [7] Bowen proved that for all non-amenable groups all Bernoulli shifts factor onto one another, improving upon partial results obtained in [28, 45]. The positive direction of Ornstein’s isomorphism theorem was extended to all countable groups through the works [16, 36, 45].

For a detailed introduction to sofic groups, see the survey [39]. Sofic entropy was introduced by Bowen [4] for actions of sofic groups admitting a generating partition.
having finite Shannon entropy. This restriction was removed, and two equivalent formulations of sofic entropy were presented in [25, 23]. Each of those three papers prove that sofic entropy is an isomorphism invariant. It is shown to extend the classical Kolmogorov–Sinai entropy of actions of amenable groups in [5, 26]. The sofic entropy of Bernoulli shifts was computed in [4, 25, 27]. Kerr proved that Bernoulli shifts over sofic groups have completely positive entropy in [24] and this was generalized to a uniform mixing condition by Tim Austin and Peter Burton [1].

Sofic entropy was computed for algebraic actions and Gaussian actions by Hayes [20, 21].

Rokhlin entropy was introduced in [41] and in [42] Rokhlin entropy is related to the open problems of classifying Bernoulli shifts, Gottschalk’s surjectivity conjecture, and Kaplansky’s direct finiteness conjecture. Naive entropy and percolation entropy are shown to be upper-bounds to Rokhlin entropy in [43] (for more on naive entropy see [10]). Sinai’s factor theorem for general countable groups is proven in [44]. In [8] Bowen proved that every free action of a non-amenable group is the factor of an action having 0 Rokhlin entropy, and that for all countably infinite groups the generic action has 0 Rokhlin entropy.

Surveys on the entropy theory for non-amenable groups can be found in [28, 9, 16].

REFERENCES

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