The 10 exam Problems: We prefer complete solutions of a few problems to many partial solutions. The value of each part of all problems is specified at the beginning.

Do each problem on a separate page. Write your name on each page. If you don’t understand some terminology, please ask. The notation throughout a given problem remains constant. Show all details and quote properly theorems you use. To pass this exam you need to collect 70 points or higher.
1 Cubics

(8 points) Show that all homogeneous cubic (degree 3) forms in 3 variables form a vector space, compute its dimension and show a basis in it.
2 Conjugacy Classes

(10 points) Let $G_1$ and $G_2$ be two finite groups with the same number of conjugacy classes and the same number of elements in each conjugacy class. Are $G_1$ and $G_2$ isomorphic? Please explain.
3 Groups of order 6

(10 points)
Describe all groups of order 6.
Hint: Use Sylow’s theorem.
4 Simple Groups

(12 points) Prove that $A_n, (n > 5)$ is a simple group.
5 Abelian groups

(10 points) Describe up to isomorphism all abelian groups of order 144.
6 The group $A_4$

(10 points) Let $A_4$ be the alternating group on 4 elements.

4 6.a) Show that $A_4$ is a solvable group.

2 6.b) Find the center of $A_4$.

4 6.c) Are there any other nonabelian groups of order 12?
7  JNF

(10 points) Let $T$ be a $3 \times 3$ matrix with complex coefficients. Describe all possible solutions of the equation $T^3 = T$.

Hint: Use JNF theorem.
8 Group Actions

(12 points) Consider the permutation action of the group $S_4$ on the 4 standard basis vectors in a complex 4 dimensional vector space. This action defines a homomorphism $\rho : S_4 \rightarrow GL(4, \mathbb{C})$.

2 8.a) Show that $\rho$ has a trivial kernel.

4 8.b) Show that there exists a vector $x$ - a common eigenvector for all images of elements of $S_4$ under $\rho$.

6 8.c) Show that the orthogonal complement to $x$ is also fixed under the action given by $\rho : S_4 \rightarrow GL(4, \mathbb{C})$. 
9 Centers

\[ 9. \text{a)} \] Find the center of the group $GL(n, C)$ for $n > 1$.

\[ 9. \text{b)} \] Find the center of the group $SL(n, C)$ for $n > 1$. 
10 Matrices

(8 points) Let $A$ and $B$ be any $n \times n$ matrices over $\mathbb{C}$. Is it possible that $ABA - BAB = E$? Please explain. Here $E$ is the identity matrix.