**Comprehensive Exam in Algebra June 2008** **NAME**__________________________________

PLEASE TRY ALL 10 PROBLEMS.

**NAME**__________________________________

G1. Let $G$ be a group and let $G = H \bowtie K$ (where $\bowtie$ is the internal direct sum). Show that if $G$ has the ascending chain condition on normal subgroups, then so does $H$. Be careful to prove all claims that you make.

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G2. A subgroup $H$ of a group $G$ is **characteristic** if $H$ is $\sigma$-invariant for all automorphisms $\sigma$ of $G$. Prove that every subgroup of a cyclic group is characteristic.

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G3. Show that no group of order $56 = 7 \cdot 2^3$ is simple. *Hint*: count elements.