Comprehensive Exam in Analysis

10–1 PM, June 18, 2007

PRINT NAME: ______________________________

- No books
- 9 problems
- 10 points for each problem
- *Show work*
- Good Luck!

SCORE:

1	________
2	________
3	________
4	________
5	________
6	________
7	________
8	________
9	________
Total	________

1. Suppose $a_n > 0$, and $\sum a_n$ diverges. Show that $\sum \frac{a_n}{1+a_n}$ diverges.

2. Let (a, b) be a nonempty open set in $\mathbb{R}$, and f be a function on (a, b). Show the following two definitions are equivalent:
(a) Let $x_0 \in (a, b)$, f is continuous at x_0 iff for any $\epsilon > 0$ there exists $\delta > 0$ such that for any $y \in (x_0 - \delta, x_0 + \delta) \cap (a, b)$, $|f(y) - f(x_0)| < \epsilon$.
(b) Let $x_0 \in (a, b)$, f is continuous at x_0 iff for any sequence $\{y_n\}_{n=1}^{\infty} \subset (a, b)$ satisfying $\lim_{n \to \infty} y_n = x_0$, $\lim_{n \to \infty} f(y_n) = f(x_0)$.