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Complex Analysis Qualifying Examination

Choose any 8 problems from 9

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1. Prove the following Jordan's lemma. Let $f(z)$ be continuos in the region $D = \{z \in \mathbf{C} : |z| \geq R_0,\ \mathrm{Im} z \geq 0\}$ and $\lim_{z \to \infty} f(z) = 0$ uniformly on D. Then for any positive number a

$$\lim_{R \to \infty} \int_{\Gamma_R} e^{iaz} f(z)\, dz = 0,$$

where Γ_R is the arc of the circle $\{z \in \mathbf{C} : |z| = R\}$, which lies in the semi-plane $\mathrm{Im} z \geq 0$.

2. Let $f(z)$ be holomorphic in the closed unit disc $\overline{D(0,1)}$. Prove

$$f(z) = \frac{1}{\pi} \int_{D(0,1)} \frac{f(w)}{(1 - z\overline{w})^2} dA(w), \quad z \in D(0,1).$$

3. Let α, β and γ are positive real numbers. Then find the radius of convergence for the series

$$\sum_{n=0}^{\infty} \frac{\alpha\,(\alpha+1)\ldots(\alpha+n-1)\,\beta\,(\beta+1)\ldots(\beta+n-1)}{n!\gamma\,(\gamma+1)\ldots(\gamma+n-1)} z^n.$$

4. Show that

$$F(z) = \int_0^1 \frac{e^{tz}}{1+t} dt$$

is holomorphic in $\mathbf{C}$.

5. Let $f : D(0,1) \to D(0,1)$ be holomorphic. Prove

$$\left|\frac{|f(z) - f(w)}{1 - f(z)\overline{f(w)}}\right| \leq \left|\frac{z - w}{1 - z\overline{w}}\right|, \quad z, w \in D(0,1).$$

6. Let $\Omega \neq \mathbf{C}$ be a simply connected domain in $\mathbf{C}$. Let $f : \Omega \to \Omega$ be a holomorphic mapping which fixes two distinct points in Ω (i.e. there are $p, q \in \Omega$ so that $f(p) = p$ and $f(q) = q$). Show that $f(z) \equiv z$ on Ω.

7. Let a be a real number, evaluate the following integral

$$\int_0^\infty \frac{\sin ax}{\sinh x} dx.$$

8. Let $f(z)$ be analytic on $\mathbf{C}\backslash\{1\}$ and have a simple pole at $z = 1$ with residue λ. Prove that for every $R > 0$,

$$\lim_{n\to+\infty} R^n \left|(-1)^n \frac{f^{(n)}(2)}{n!} - \lambda\right| = 0.$$

9. Suppose that $f(z)$ is an entire function such that

$$|f(z)| \leq Be^{A|z|}, \quad z \in \mathbf{C}$$

for some positive numbers A, B. Let $\omega_1, \omega_2, \cdots$ be the zeros of f listed with appropriate multiplicity. Prove that

$$\sum_{n=1}^{\infty}(1+|\omega_n|)^{-\alpha} < \infty$$

for all $\alpha > 1$.