COMPLEX ANALYSIS

Qualifying Exam

Wednesday, June 18, 2008 – 10:00-12:30, MSTB 114

Problem	1	2	3	4	5	6	7	8	9	Σ
Points										

Student's name:

Problem 1.

Find explicitly a conformal mapping of the domain

$$\{z \in \mathbb{C} \mid |z| > 1, \ \ \operatorname{Re} z > 0, \ \operatorname{Im} z > 0\}$$

to the unit disc.

Problem 2.

Let $f(z)$ be entire holomorphic function on $\mathbf{C}$ such that $|f(z)| \leq |\cos z|$. Prove $f(z) = c \cos z$ for some constant c.

Problem 3.

Show that there is a holomorphic function defined in the set

$$\Omega = \{z \in \mathbb{C} \mid |z| > 4\}$$

whose derivative is

$$\frac{z}{(z-1)(z-2)(z-3)}.$$

Is there a holomorphic function on Ω whose derivative is

$$\frac{z^2}{(z-1)(z-2)(z-3)}?$$

Problem 4.

Denote by D the unit disk, $D = \{z \in \mathbb{C} \mid |z| < 1\}$. Does there exists a holomorphic function $f : D \to D$ with $f\left(\frac{1}{2}\right) = \frac{3}{4}$ and $f'\left(\frac{1}{2}\right) = \frac{2}{3}$?

Problem 5.

Evaluate the improper integral

$$\int_{-\infty}^{+\infty} \frac{x^2 \sin(\pi x)}{x^3 - 1}\, dx$$

Problem 6.

Prove that the product $\prod_{k=1}^{\infty}\left(\frac{z^n}{n!}+\exp\left(\frac{z}{2^n}\right)\right)$ converges uniformly on compact sets to an entire function.

Problem 7.

Let F be a family of holomorphic functions on the unit disc $D(0,1)$ such that each $f \in F$ satisfying

$$|f(0)|^2 + \int_{D(0,1)} |f'(z)|^2 dA(z) \leq 1$$

Prove that F is a normal family on $D(0,1)$.

Problem 8.

Let f be holomorphic on the upper half plane U and continuous on $U \cup [0, 1]$. Assume that

$$f(x) = x^2 - x + 1, \quad x \in (0, 1)$$

Find all such functions f.

Problem 9.

Show that there is no holomorphic function $f(z)$ on $\{z \in \mathbb{C} \mid 1 < |z| < 3\}$ satisfying

$$\left| \frac{f(z)^2}{z} - 1 \right| < 1.$$