Math 77A Midterm (Due February 21)

Instructions: The responses to Problems 1 and 2 can be handwritten and turned in at the beginning of lab on Feb. 21. Problems 3 and 4 will require some MATLAB code. Please submit your code as an `.m` file named `midterm_yourlastname.m` by emailing it to `eesser@uci.edu`. If you are submitting multiple files, please zip them together in a file named `midterm_yourlastname.zip`.

1. Suppose we sample a sine wave $s(t) = \sin(2\pi 199t)$ at $t = n\triangle$ where the sampling interval $\triangle = .01$ and $n = 0, 1, 2, 3....$

1a. What is the sampling rate?

1b. What is the Nyquist Frequency corresponding to this sampling rate?

1c. What is the frequency v with smallest magnitude such that $\sin(2\pi vt)$ agrees with $s(t)$ at the sampled points? Explain.

1d. For a signal such as $s(t)$ with no frequencies higher than 199 hertz, what is the Nyquist Rate?

1e. Summarize the Nyquist-Shannon Sampling Theorem in your own words.

2. Calculate the Discrete Time Fourier Transform of a rectangular window defined by

$$w_n = \begin{cases} 1 & \text{if } n \in [0, L-1] \\ 0 & \text{otherwise} \end{cases} \qquad \text{for integer } n$$

2a. The DTFT of w_n is defined by

$$W(\omega) = \sum_{n=-\infty}^{\infty} w_n e^{-i\omega n} \ ,$$

which for this choice of w_n reduces to $\sum_{n=0}^{L-1} e^{-i\omega n}$. Show that

$$W(\omega) = e^{-i\omega \frac{(L-1)}{2}} \frac{\sin(\frac{\omega L}{2})}{\sin(\frac{\omega}{2})} \ .$$

2b. Draw a rough sketch of the absolute value of $W(\omega)$.

2c. Find the zeros of $W(\omega)$ nearest to the origin.

2d. How is the main lobe width related to L?

2e. Does the relation in 2d obey the uncertainty principle? Explain.

3. In MATLAB, define as follows a triangle wave that is 2 seconds long, has a frequency of 5 hertz and is sampled at a rate of 100 samples per second.

```
v = 5; T = 2; fs = 100; t = 0:1/fs:T-1/fs;
x = 2*abs(2*(v*t - floor(v*t+1/2))) - 1;
```

Also define a sine wave of the same frequency and having the same length.

3a. Use MATLAB's `filter` function to define a filter according to

$$y_n = \frac{1}{4}(x_n - 2x_{n-1} + x_{n-2})$$

Plot the first 10 elements of the impulse response for this filter.

3b. Plot a sampled version of the frequency response by computing the absolute value of the DFT of a length $N = 200$ impulse response. Is this a high pass or a low pass filter?

3c. In the same MATLAB figure, plot the magnitude of the DFTs of the sine wave and the triangle wave.

3d. Apply the filter from 3a to both waves and plot the filtered results in the same figure.

3e. Design a low pass filter so that the filtered version of the triangle wave looks similar to the sine wave. Plot the filtered triangle wave and the original sine wave in the same figure.

4. Load khan.wav from the course website. Design an ideal band pass filter in the frequency domain that keeps the frequencies between 100 and 1000 hertz, discarding all others. Plot the magnitude of the DFT coefficients of the filtered khan signal as a function of frequency in hertz.