# Aggression

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**Abstract**

Aggression is an unsolved game in which no previous research has been conducted. We have analyzed the game to find the optimal strategy. It is a Risk-like game with two players in which territories are drawn, occupied, and then invaded. Through the use of an adaptive learning program and basic analysis of strategies, we have found a partial solution with pre-set troops on an simplified $4 \times 4$ game board. We have found an optimal solution for the final game phase.

## 1 Introduction

Aggression is a Risk-like game with 3 distinct stages involving 2 players. Little or no previous research of Aggression exists so the optimal strategy of this game has yet to be found. The primary focus of this research is to find the optimal strategy in the game. Due to the complexity of the 3 phases and the numerous possible game states at each phase, we have decided, explained further in Section 5, to use an simplified game board and look at the final phase in detail and apply basic strategies to that one phase. To tackle this problem, we have written an adaptive learning program to efficiently search for optimal strategies, as talked about in Section 6.

In this paper, we discuss a variety of basic strategies in Section 5 that we have looked at and analyzed. We also look into the 3 different phases and the strategies pertaining to each specific phase as well as provide suggestions on how to play a certain phase. In Section 6, we take a closer look at our adaptive learning program and how these help support our proposed solution.

## 2 Background

Aggression is a 2 player Risk-like game with 3 phases:

1. Drawing of regions,
2. Occupying of regions with troops, and
3. Invasion of regions by opposing troops.

**Phase 1: Drawing of Regions**

- The starting player starts by drawing a loop as shown in Figure 1.

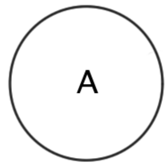


Figure 1: Player 1's first move

- Players then simplified drawing regions until 20 regions have been drawn.
- Regions must be drawn off each other as depicted in Figure 2.

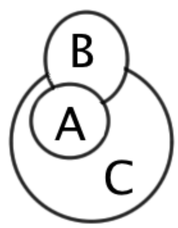


Figure 2: Letters show order of regions being drawn

- Where the regions touch must be clearly emphasized (i.e. cannot have unclear boundaries) as shown in Figure 3.

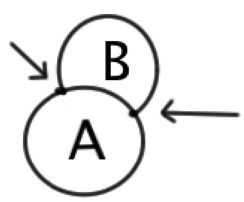


Figure 3: Letters show order of regions being drawn

- Otherwise, countries can be drawn in any shape or position.

**Phase 2: Occupying of Regions**

- Players have 100 troops each.
- Players alternate placing any number of troops in one not occupied by opponent.

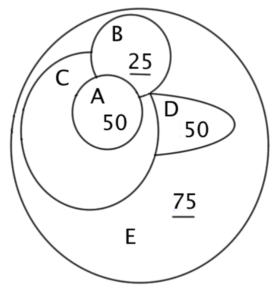


Figure 4: not underlined = player 1, underlined = player 2

- Notice that players do not have to place troops in regions next to each other.
- The number of troops do not have to be divided as evenly as Figure 4 depicts above.
- If a player finishes allocating troops before the other player, the player with troops remaining does not have to use all remaining troops in one turn but can take as many turns necessary to finish using up all 100 troops, making it an advantage to finish after the other player.
- All regions do not need to be occupied as displayed in Figure 5.

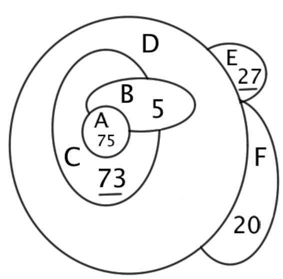


Figure 5: It is alright to leave region D unoccupied.

**Phase 3: Invading of Regions** After phase 2, the allocating of troops, has been completed, players then take turns crossing out each others troops. The following lists the criteria necessary to invade an opposing region:

- A player can only invade if the opponent's region is adjacent to the region the player is attacking from.

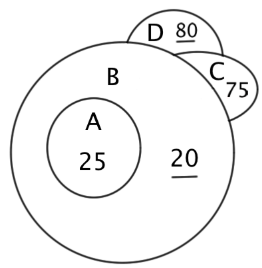


Figure 6: A can invade B but cannot invade D.

- A player can only invade if he has more troops than the opponent in said region.

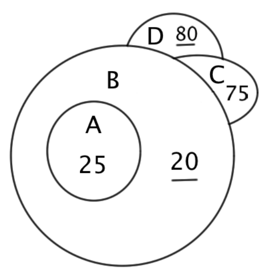


Figure 7: A can invade B but C cannot invade D.

- At the end, the winner is determined based on which player has the most regions after the cross-out phase (phase 3), not based on how many troops outstanding.

# 3 Combinatorics

In the game Aggression, players simplified drawing the board in any shape they choose, but must start with a loop. Then each player proceeds to add regions with at least two connecting points on an existing region. For our analysis we have focused on the $1 \times N$ and $N \times N$ boards.

**Troop Placement and Possible Boards** The total number of boards in Aggression can be calculated using compositions and combinations dependent on the number of regions and troops, regardless of the configuration of the board. A composition is the calculation of the number of ways any nonzero number $p$ can be divided into $l$ parts in with order matters and such that the sum of these parts add up to $p$. Let $n$ represent the total number of troops allocated to each player and let $r$ denote the total number of regions on the board. Consider $k$, the number of sections a player can divide their troops into on the board. Using compositions to determine how the $n$ amount of troops can be divided accounts for the order that the troops are placed and therefore gives the total number of boards. Let $\mathcal{C}(n,k)$ denote the composition of troops into sections for player 1 and $c(r,k)$ represent the combination of regions and parts. Similarly let $\mathcal{C}(n,r-k)$ composition of the number of troops for player 2, into $r-k$ regions i.e. the regions that remain after player 1 has distributed all of their troops. Therefore the equation below can be used to determine the total number of possible boards for any number of regions and any number of troops.

$$\sum_{k=1}^{r-1} \mathcal{C}(n,k) \cdot c(r,k) \cdot \mathcal{C}(n,r-k). \qquad (1)$$

$4 \times 4$ **Grid Board** The board we have decided to study most closely is a game board in the shape of a $4 \times 4$ grid. For this specific board, the number of troops, $n$, is 100 and the total number of regions, $r$ is 16. Equation 1 can be used to analyze the total number of possible boards that can be created on this board and we obtain

$$\sum_{k=1}^{15} \mathcal{C}(100,k) \cdot c(16,k) \cdot \mathcal{C}(100,16-k) = 6.88E+32. \qquad (2)$$

**Number of Possible Boards** The number of possible boards drawn can be obtained by $2^{\frac{n^2}{2}}$. As you can see, this escalates quickly, leading to an overwhelmingly large amount of possible boards just for phase 1. Please see table 1.

# 4 Approach

Little is known about Aggression due to no previously conducted research on the game. Because of that, our goal is to analyze Aggression and find the optimal strategy for the game. This includes:

- Best way to draw regions
- Best way to place troops

- Best way to invade

For simplicity's sake, we chose to begin our analysis of Aggression by considering a $4 \times 4$ game board. From there we found that we must work backwards from the last phase to find the optimal strategy.

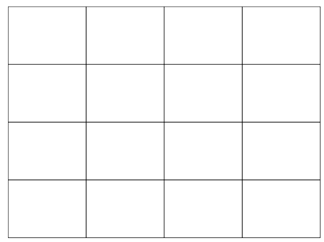

Figure 8: $4 \times 4$ matrix game board

## 5 Basic Strategy

We looked at several basic strategies for the second phase of the game.

**Priority Spacing** We first looked at the priority spacing, how the location and type of space should be treated, then identified the 3 different types of spaces on the game board.

1. Edge
2. Corner
3. Center

Based on the different characteristics of these 3 spaces, the spaces can be prioritized in different ways according to location of space and amount of troops remaining. Figure 9 shows these spaces on our game board.

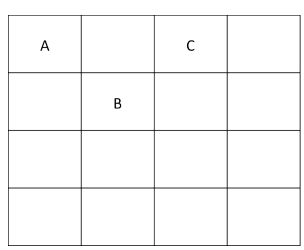


Figure 9: A = corner, B = edge, C = center

**Vulnerability** The vulnerability of a certain space determines how easily attacked by other regions a space is. The 3 different types of spaces also demonstrate varying levels of vulnerability. Looking back up at Figure 9, we can see that the

1. edge, space B in Figure 9, shares 3 sides with other regions, making it more vulnerable than space A but less the space C,
2. corner, space A, shares only 2 sides with other regions, making it the least vulnerable space on the matrix, and
3. center, space C, is surrounded on all sides, making it the most vulnerable space.

**Troop Placement** The placement of troops refers to where and how many troops a player should place in a certain type of space. Taking into account the 2 strategies mentioned above, we can offer suggestions as to roughly how many troops should be placed in a particular type of space.

1. For edges, we suggest placing a medium amount of troops in the space due to middle vulnerability and it's exposure to sharing boundaries with 3 other countries.
2. For corners, we suggest placing a smaller amount of troops in these spaces because it is nicely located away from most other countries, only sharing 2 boundaries with other regions.
3. For center pieces, we suggest placing the majority of your troops into these spaces because there is a higher percentage of being invaded as well as invading.

Through the use of an adaptive learning program and other basic programs, we have found a partial solution. This partial solution involves having solved pre-set boards, meaning we have eliminated the first 2 phases of the game and focused on the cross-out phase.

**Cross-Out Strategy** Another type of basic strategy we have analyzed involves the prioritizing of the invasion of troops. First off, we suggest crossing out the region in which your opponent has the most troops placed possible. In Figure 10, you can see that player one would want to use his 32 cross out player two's 25 first.

**Mirror Strategy** This strategy focuses on phase 2, the implementing of troops into their respective regions. For example, if we look at a $4 \times 4$ board, player 2 need only mirror the position and amount of troops player 1 puts down. This often leads to a a tie instead of a loss which is the norm, in this case, for player 2, shown in Figure 11.

| | | | |
|---|---|---|---|
| 11 | | $\underline{2}$ | $\underline{3}$ |
| $\underline{1}$ | $\underline{32}$ | 25 | |
| $\underline{9}$ | 80 | | $\underline{31}$ |
| 21 | $\underline{22}$ | 19 | 4 |

Figure 10: Player blue's 32 cross-outs Red's 25.

| | | | |
|---|---|---|---|
| 25 | 3 | 12 | 1 |
| | 40 | $\underline{1}$ | $\underline{18}$ |
| 18 | 1 | $\underline{40}$ | |
| $\underline{1}$ | $\underline{12}$ | $\underline{3}$ | $\underline{25}$ |

Figure 11: Player blue mirror exactly player red's moves.

# 6 Adaptive Program and AI

We created an adaptive learning program and implemented some of the basic strategies presented in the paper above. We used 2 pre-set $4 \times 4$ boards to test the cross-out strategy. Finally, we wrote an adaptive learning of a pre-set $2 \times 2$ and $4 \times 4$ game board. The result of the $2 \times 2$ shows that in that particular game board, shown in Figure 12, that it is always a tie.

| | |
|---|---|
| $\underline{1}$ | 2 |
| 1 | $\underline{2}$ |

Figure 12: solved pre-set $2 * 2$.

In the case of the pre-set $4 \times 4$, depicted in Figure 13 below, the result is...

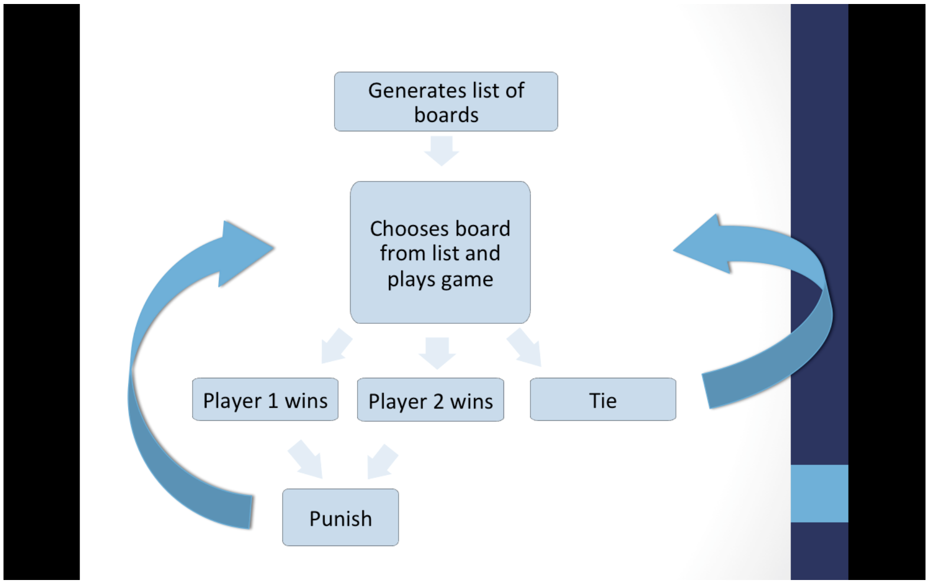


Figure 13: Adaptive Learning Flowchart

# 7 Conclusion

We have found a partial solution to our problem of finding an optimal strategy of Aggression and its 3 phases. Through the analysis of basic strategies and the adaptive learning, we have solved a pre-set $2 \times 2$ and $4 \times 4$.

The next step in finding the optimal strategy would be to analyze more than 1 phase at a time and develop a more efficient adaptive learning program to better solve the game. That would give us a better data to analyze, leading us to finding the optimal strategy for all phases of the game for all board types.

Table 1: Troop Placement Combinatorics

| | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 10368 | 2395008 | 2395008 | 2395008 | | | | | | |
| 50 | 11063808 | 121159761408 | 267604473024000 | 2.45E+17 | 1.19E+20 | 3.53E+22 | 6.90E+24 | 9.45E+26 | 9.41E+28 | 7.04E+30 |
| 100 | 188257608 | 8820057192408 | 8.51E+16 | 3.48E+20 | 7.75E+23 | 1.08E+27 | 1.02E+30 | 6.88E+32 | 3.48E+35 | 1.36E+38 |