Using the Hawk-Dove Model and Ordinary Differential Equation Systems to Study Asian Carp Invasion

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Abstract

Asian carp have invaded the Mississippi and Illinois Rivers, and they are currently threatening the Great Lakes. Their invasion can alter the food web, hence negatively impacting these ecosystems. Electric fences or targeted removal, if implemented correctly, can economically control the invasion. By controlling or preventing the invasion, we can prevent further disruption to these ecosystems.

1 Introduction

Bighead and silver carp, also known as Asian carp, are large filter-feeding fish native to China. Adults are approximately 20 kilograms in weight and 900 millimeters in length. Their diet consists of plankton, and they consume 6.6-11.3% per day of their body weight, which is a large amount. Asian carp were introduced to the United States in the 1970s to control the phytoplankton population in commercial ponds. They escaped and migrated into the Mississippi and Illinois Rivers and are currently threatening to invade the Great Lakes [2]. If the invasion is not controlled, then the carp will compete with native fish for food, which can alter the food chain, hence negatively impacting the ecosystem.

In order to control the Asian carp invasion, various methods, such as electric fences and targeted removal, are used. There is currently one electric fence between the Upper Mississippi River System (UMRS) and the Great Lakes. A study conducted by the Illinois Natural History Survey discovered that electric fences, along with an acoustic bubble current system, is 83% effective in preventing bighead carp from passing. However, the barrier is not as effective in constraining younger carp. Another concern is that silver carp may be able to jump through the barrier if startled. It costs approximately $4 million just for the demonstration barrier; the total cost for the electric barrier is $16 million. Targeted removal is the process of removing Asian carp using nets or Rotenone, a chemical extracted from derris and cube root. Areas that are most likely to be subject to selective removal include wastewater treatment plant outfalls and areas near dams. The cost of selective removal is $2 million [2].

In this paper, we will approach the Asian carp problem mathematically, using a game-theoretical model and ordinary differential equation (ODE) systems along with a diffusion model to study the speed of the invasion. Then we will alter the model to add an electric fence or targeted removal.

2 Our Approach

2.1 Hawk-Dove Model

To model the invasion, we used the Hawk-Dove model. The Hawk-Dove model is an application of game theory used to study animal behavior. There exists two strategies: Hawk and Dove. Hawks attack their opponents; Doves retreat if their opponent is a Hawk and share the victory if the opponent is another Dove.

	Hawk	Dove
Hawk	$\frac{V-C}{2}$	V
Dove	0	$\frac{V}{2}$

Table 1: HD payoff matrix

Table 1 shows the payoff matrix of a Hawk-Dove game. V and C are constants that stand for the victory payoff and the cost of injury in a conflict, respectively. If two hawks encounter each other, then they both obtain some victory, but there is a cost due to injury. If a hawk encounters a dove, then the dove retreats; hence the hawk will obtain all the victory. If two doves encounter each other, then they will evenly split the victory. Hawk is the optimal strategy if $V > C$. If $V < C$, then the optimal strategy is to play Hawk $\frac{V}{C}$ times [4].

We will use the payoff matrix above for our model, except that we choose $V = 2$ and $C = 4$. Let A be our new payoff matrix; in other words, $A = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$. We chose these values for our V and C is because we want the carrying capacity $\frac{V}{C} = 0.5$; based on data from the Long-term Illinois River Fish Population Monitoring Program (LTRMP), the percentage of silver carp captured compared to the other species of fish was 50% in 2008 [1]. In order to represent a change in population fractions over time, we also used the normalized ODE system $\mathbf{y\prime} = \frac{\mathbf{y_i(Ay_i - y^T Ay)}}{|\mathbf{y^T Ay}|}$, where $\mathbf{A}$ is the payoff matrix, and $\mathbf{y}$ is a vector representation of population fractions. Normalizing the system will give results with respect to the average fitness rather than the absolute fitness. We chose the initial time unit to be 0 and the final to be 10.

2.2 Diffusion Model

The Hawk-Dove ODE is not informative for a partial investigation of the asian carp invasion. In order to model fish movement and to add a spatial component, we modeled the river as line of n cells and used a one dimensional reaction-diffusion formula:

$$\frac{\partial w}{\partial t} = F(w) + D\Delta w, \tag{1}$$

[3] where:

- F a $2n \times 1$ vector representing the change of population fractions over time. In other words, a slightly modified version of our Hawk-Dove ODE
- w is the $2n \times 1$ vector representing the population fractions (similar to $\mathbf{y}$ in the Hawk-Dove ODE)
- $D\Delta$ is a $2n \times 2n$ matrix $\begin{pmatrix} \Delta_{WH} & 0 \\ 0 & \Delta_{WD} \end{pmatrix}$ where Δ_{WH} and Δ_{WD} are discrete Laplacian matrices. In our case, we will assume that $\Delta_{WH} = \Delta_{WD}$. So $\Delta_{WH} = -E^T W E$, where E $= \begin{pmatrix} -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{pmatrix}$, an $(n-1) \times n$ edge-node adjacency matrix chosen to model the concept that fish from a cell of higher concentration will want to migrate to a cell of lower concentration, and W is the $(n-1) \times (n-1)$ diagonal matrix of diffusion coefficients. Each cell represents one kilometer of the entire river.
- t represents the time period. In ODE system, one unit of time is chosen automatically by our program.

For our model, we chose W $= \begin{pmatrix} 0.1 & & \\ & \ddots & \\ & & 0.1 \end{pmatrix}$, since Asian carp migrate an average of 10 kilometers a day [2], which implies we want to set a higher diffusion rate. We chose to divide the river into 10 cells; in other words, we chose $n = 10$ Also, we chose random time step, which is generated randomly by our program in the Matlab.

In order to determine optimal methods to control the Asian carp invasion, we altered the diffusion model to add an electric fence or targeted removal. Because the electric fence effectively prevents most carp from migrating to the other side, we changed certain diagonal entries of W from 0.1 to 0.000001, a much lower diffusion rate. To model targeted removal, we altered the top row of the payoff matrix A by adding another matrix in the form $\begin{pmatrix} -5 & -0.75 \\ 0 & 0 \end{pmatrix}$. The reason why we only alter only the top row is because we are only targeting Asian carp; hence their fitness is negatively impacted. Targeted removal does not affect the fitness of the native fish, which is why the bottom row contains only zeros.

Note that for our model to work, we must make some assumptions:

1. All Asian carp act like Hawks, and all native fish act like Doves.
2. Every fish in the river is either an Asian carp or a native species.
3. Biomass is conserved.
4. During targeted removal, biomass is still conserved due to reproduction.
5. The flow, temperature, food supply, and other environmental conditions are the same for every cell in the river.
6. Fish dispersal is independent of flow, temperature, and other environmental factors.
7. The carrying capacity is constant.

3 Results

We chose two different initial conditions, ran the model without treatment, and then experimented with parameters for the electric fence and targeted removal:

1. **Beginning of an invasion:**

   For our initial condition, we set some amount of carp in the first 3 cells (25% in cell 1, 10% in cell 2, and 5% in cell 3); the rest of the cells contain no carp.

   The graph above shows the final population fractions of Asian carp in each cell after no treatment, after implementing an electric fence after cell 3, and after selectively removing Asian carp from cells 4-7. Our simulations show that the best place to put an electric fence is the space between cell 3 and cell 4. At the same time, the best fishing strategy, given at most one set of targeted fishing per each cell is to fish through cell 4 to cell 7.

   If there is no treatment, the total population fraction of Asian Carp in cell 4-7 will be 3.5511, while putting an electric fence, 1.5023, and fishing through cell 4-7,1.6327.

2. **During an invasion:**

   For this case, each cell contains some amount of Asian carp. Most cells contains at least 20% carp.

   An interesting result is that for the second initial condition, the electric fence does not make a difference. Hence, the best solution is just to use targeted removal.

   The Average invasion index represents the total population fractions of asian carps in ten cells after certain implementation. Here, the average invasion index is the total population fractions of asian carps over 20 random initial population of asian carps.

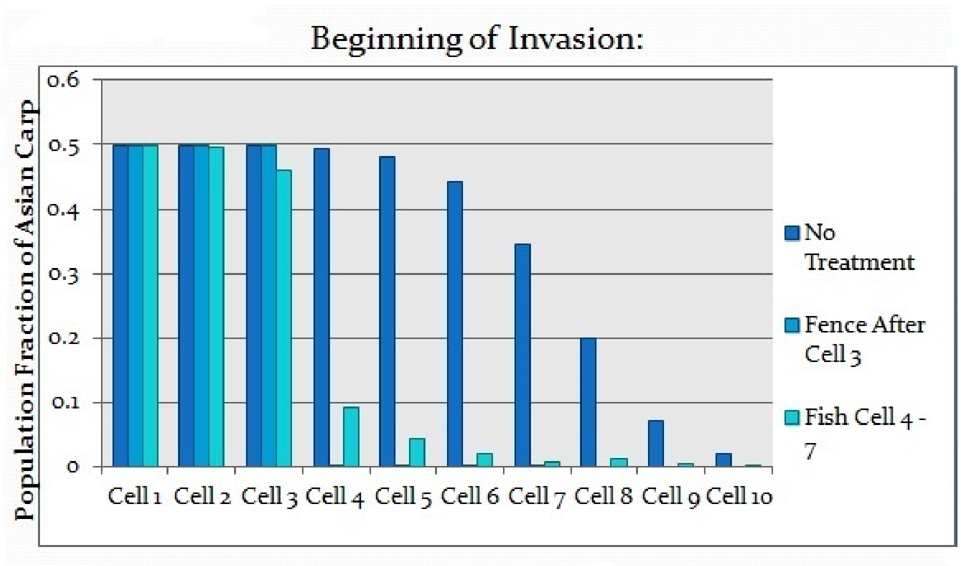

Figure 1: Results for the Total Population of Asian Carps After Different Treatments at the Beginning of an Invasion.

Given two sets of targeted fishing, we get the result as shown in the graph, with the x-axis representing where we place the first set of targeted fishing and y-axis representing the other. The color bar shows the invasion index, meaning the higher the invasion index is, the redder the color.

As the graph shows, placing the two targeted fishing in the same cell leads to a high invasion index, which suggests that this strategy is bad.

4 Discussion

- Beginning of an Invasion:

  If targeted removal is as effective as our assumption, with the given initial Asian Carp population fractions (Cell 1: 025, Cell2: 0.1, Cell3: 0.05), then the optimal strategies are to construct an electric fence after cell 3 or to remove carp from cells 4-7. However, due to the high cost of the electric fence, it is more money-efficient to fish from cells 4-7. In shorts, the best strategy is to fish through from cell 4 to cell 7.

- During an Invasion:

  Given certain amount of targeted fishing groups, putting all of the targeted fishing groups in the same cell will lead to a relatively high invasion index compared to other possible strategies of the same amount of targeted

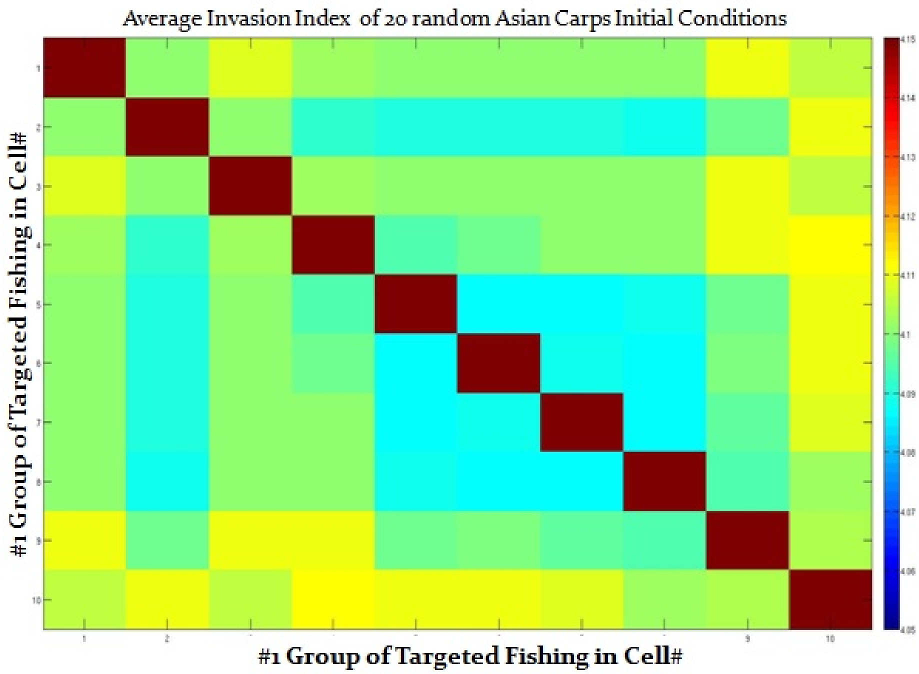

Figure 2: Results for Population Fractions of Asian Carps After Placing Two Targeted Fishing in Different Cell.

fishing sets. In other words, placing all the available fishing resources in the same cell is a bad strategy.

With the current 20 random initial Asian Carps population iterations, and given two groups of targeted fishing: our results suggest that placing the two fishing groups in separate cells between the center and end of the invasion domain is a good strategy, where we observe a relatively low invasion index.

5 Limitations

Like most mathematical models, ours has some limitations:

1. Native and invasive fish interactions are most likely more complicated than that represented in the Hawk-Dove model.
2. Most likely, there will be a change in biomass. For example, Asian carp can deplete the food supply, thus eventually reducing total population.

3. In addition to fish dispersal, fish exhibit active movement towards food sources and other favorable environmental conditions.

6 Future Work

For our future work, we plan to add a Retaliator, which will represent a native species which shares food with other native species and fights for food against Asian carp, to our Hawk-Dove model. In addition, we will incorporate a term for active movement of fish, and to reassess our results for later time points.

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