

Disjoint Set Forests for Maintaining Connected Components

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Motivation: Uniquely Labeling Connected Components

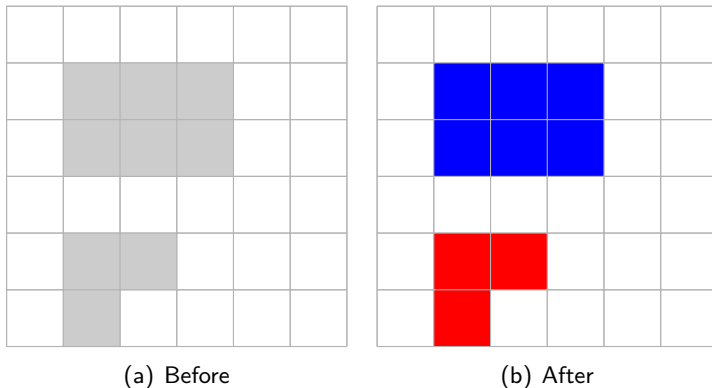


Figure: Unique labels of the connected components of a graph

For each pair of vertices, if the pair is connected, union their components.

Disjoint-Set data structure for keeping tracking of partitions of a set. Given a partition of a set of elements, solve two problems dynamically:

- Find(x) Determine the subset for which an element belongs to
- Union(S_1, S_2) Merge two subsets into a single set

Definition

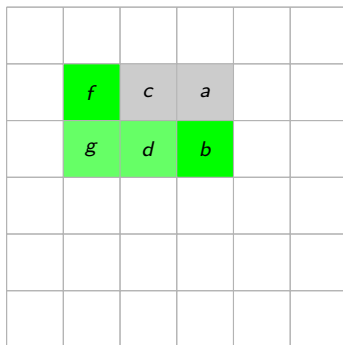
For undirected graphs, vertex v is *reachable* from vertex u if there exists a path from u to v . A path from u to v is denoted as $u \rightsquigarrow v$.

Definition

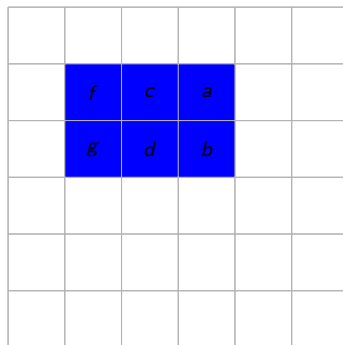
A *connected component* of an undirected graph is a subgraph such that any vertex of this component can reach every other vertex and contains no other vertices of the supergraph.

Note that connected components are a type of equivalence class.

Examples



(a) $f \rightsquigarrow b$



(b) $\text{equiv}(f) = \{c, a, g, d, b\}$

General Algorithm for Connected Components

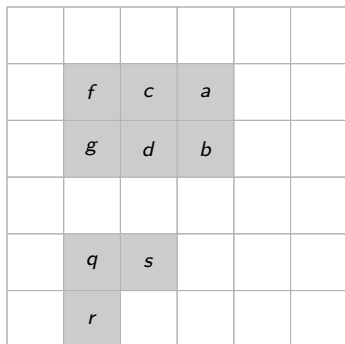
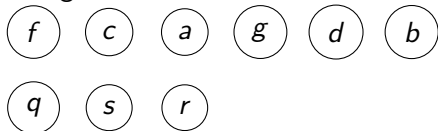


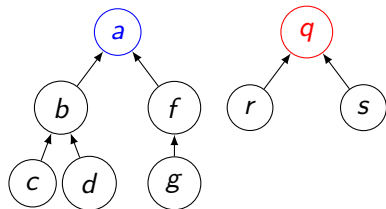
Figure: Reference Graph

- Reduce the graph into a set of sets where each set contains only an individual element. Blank spaces are ignored.



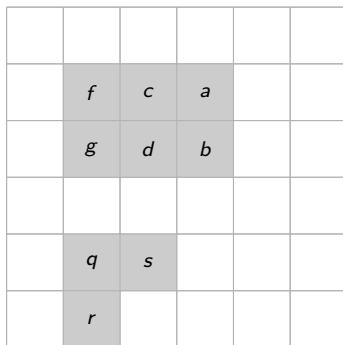
- The second step is to merge sets if their components are adjacent to each other.
- We require a data structure to maintain connected components.

Union-Find Pseudocode

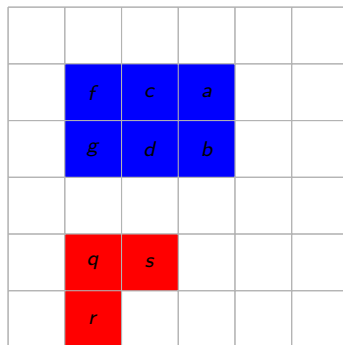


```
procedure FIND( $x$ )  
  if  $x$ .parent  $\neq x$  then  
     $x$ .parent  $\leftarrow$  Find( $x$ .parent)  
  end if  
  return  $x$ .parent  
end procedure  
procedure UNION( $x$ ,  $y$ )  
  Find( $x$ ).parent  $\leftarrow$  Find( $y$ )  
end procedure
```


Labeling Components with Disjoint Sets



(a) Before



(b) After

Thank you