Rock Paper Scissors Hex

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1 Abstract

The basis of our results consisted of pinpointing winning strategies for the game Rock Paper Scissors Hex which implements the gameplay of Rock Paper Scissors into the board of Hex. Analysis of this combinatorial game relates to chess, however it has less variety of game pieces. We created an adaptive learning program to help identify the winning strategies for the game. It was found Player 1 has the winning strategy for all game boards studied. Therefore the game is unfair.

2 Introduction

Rock Paper Scissors Hex is a zero-sum, asymmetric, normal combinatorial game which consists of two players making moves sequentially on a board made of hexagons. The number of tokens each player can have varies according to the width of the desired game board. As board games expand, additional tokens continue the pattern of the former three tokens. For example, a 3x3 board contains Rock, Paper, and Scissors, respectively, on the top and bottom rows of the board (Figure 1), while a 3x5 board consists of Rock, Paper, Scissors, Rock, Paper on the top and bottom rows of the board.

Hex 3x3 Game Board.

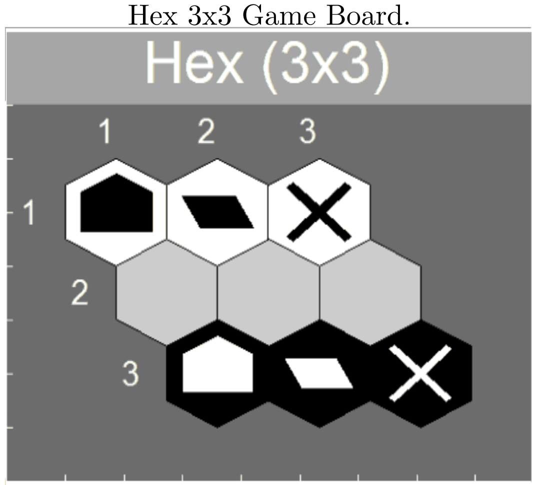

Figure 1: From left to right: Rock, Paper, Scissors. Player 1 starts on top as White, while Player 2 starts on bottom as Black.

Gameplay consists of two players who alternate turns moving one of their own tokens on the board. Legal moves are determined by moving to any available adjacent hexagon (as seen in Figure 2). However, the objective of the game is to capture all of your opponent's tokens or reach the opponent's side of the board without getting captured. Rules for capturing opponent tokens are as follows:

- Paper captures Rock.
- Scissors captures Paper.
- Rock captures Scissors.
- Identical tokens of opposing players cannot capture one another.

When Player 1's Paper is adjacent to Player 2's Rock, and it is Player 1's move, it has the ability to replace Rock with Paper. Similarly, when Scissors is adjacent to Paper, Paper is removed from the board, and replaced with Scissors just as in Chess.

Hex 3x4 Game Board

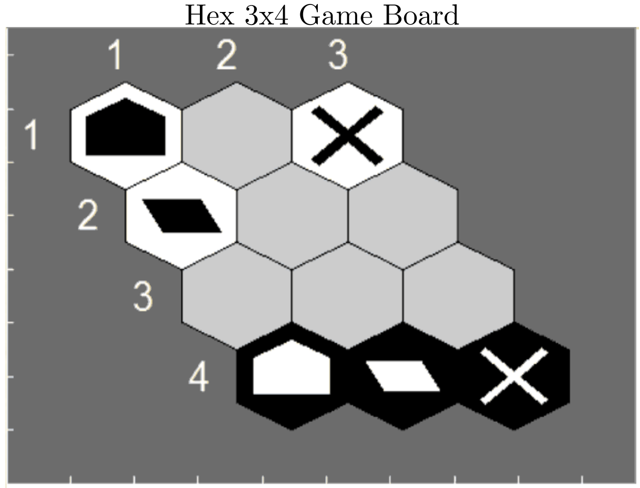

Figure 2: Top white player moving paper to an adjacent hexagon.

3 Combinatorics

We found it relevant to use the Permutation Formula in order to find the total number of possible starting game boards.

$$P = \frac{n!}{(n-k)!} \rightarrow \frac{3!}{(3-3)!} = 6$$

Where n represents the number of objects to choose from, and k represents the amount of objects that are picked out of the entire pile. For example, on a 3x3 game board, we have 3 available tokens (This is the n value). From those 3, we want to choose all 3 of those tokens (which becomes k) to find the total number of ways to arrange them. Our result is 6. However, this only gives us the rearrangement of tokens for one side of the board. Thus, this computation is necessary for both sides of the board. Repeating the process and multiplying the top and bottom results together yields 36.

We were also able to calculate the total possible first moves that player 1 can make, which is 5. For one game board, there are 5 new game boards; meaning, for 36 initial game boards, there are a total of 180 possible opening moves for player 1. However, because Player 1 can only play on one board and not 36, this made us narrow down our research to only 1 board. (Figure 1)

Within this board, the top player is guaranteed to win, regardless of the width of the board. Through our theorems, it can be deducted that the optimal strategy is to move certain tokens to specific locations in order to avoid wasting moves, and losing tokens unnecessarily. This analysis shows us that the game is not fair, allowing us to determine who has a winning strategy and why.

4 Adaptive Learning

We created an adaptive learning program to aid us in finding winning strategies, the average amount of moves that it takes to win, and the time frame that it takes it to find the winning strategy. In this program, the computer generates all of the available moves continuously without making any illegal moves (moves which are not to adjacent available positions, or capturing pieces illegally). The computer then randomly chooses one move out of the pool of possible game states that it can create from each game board and repeats the process until a winner can be declared. The computer can be prompted to run any amount of trials desired.

For example, when set to run 5,000 trials on a 3x3 game board, the computer runs as many trials as necessary until it determines the winning strategy. If the computer cannot determine who has the winning strategy by the end of 5,000 trials, the program will not yield results of winning strategies.

This means that it is necessary to run a greater amount of trials then was prompted originally in order to find the winning strategy for the board size at hand.

The Adaptive Learning Algorithm finds a winning strategy based on the following method:

- Potential game boards are weighted with a value of 1.
- Losing moves are progressively adopted into a non-reusable group of game states
  - Meaning the losing move is punished by weighing it with a value of 0
- After this occurs over a large number of games, one of the two computers (whoever is losing) will essentially run out of moves and have to resign in certain game states.
- The computers continue to play until either:
  - One computer resigns instantaneously on its first move due to all of its moves being punished.
  - The amount of trials given for a particular game board are not sufficient to determine who has a winning strategy.

5 Theorems

Based on our research, we have proven that all of these theorems are true.

- The optimal strategy for the top player on a 3x3 board, as seen in Figure 1, is to move paper, the middle piece, forward one row and to the left.

- No matter which token the bottom player decides to move, the top player will always be able to make the next game state a winning game. This holds regardless of whether the top player on the board makes the opening move or the second move of the game.

Hex 3x3 Game Board.

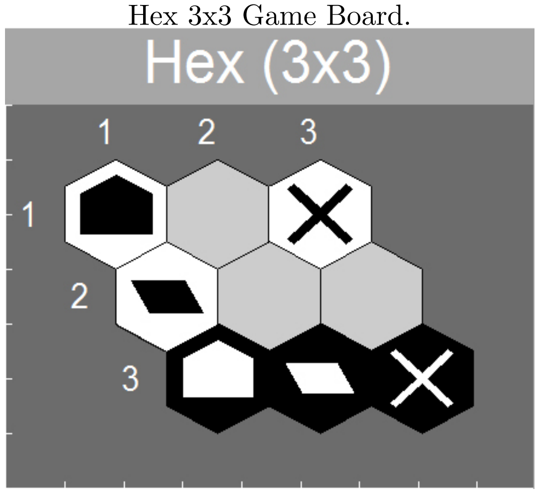

Figure 3: No matter what the bottom player does next, the top player will always win.

- Alternatively, another optimal strategy for the top player is to move Scissors, the far right token, to the 2nd row and 3rd column.
  - Due to rock being on the opposite side of the board, the black [bottom] Rock token cannot capture the white [top] Scissors nor the other side of the board before Scissors of the top player reaches the paper of the player on the bottom row.

Hex 3x3 Game Board.

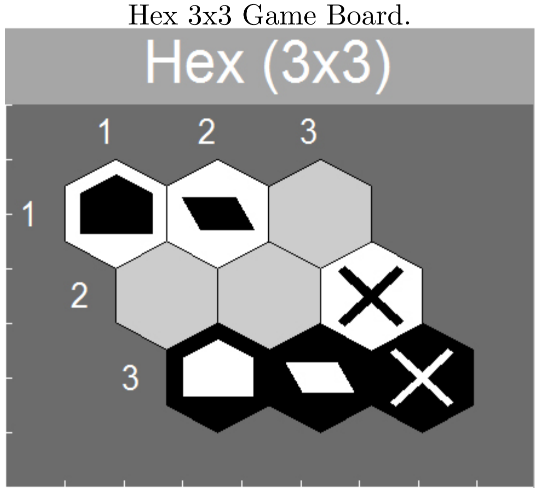

Figure 4: Scissors being moved to the right.

- The optimal strategy for the top player is to move paper along the left side of the game board, as shown in Figure 2.
  - The minimal amount of moves to win in a 3x4 case consists of Player 1 moving Paper to the left and continuing forward along the left side of the board. Because the only token that can beat Paper is Scissors, and Scissors resides on the opposite side of the board, it is impossible to prevent Player 1 from reaching the opposite side of the game board. Therefore, Player 1 possesses the winning strategy of moving paper to the previously stated position.

6 Future Work

Although, our adaptive learning program struggles to find a result with boards that consist of a height greater than 3, we were still able to find a winning strategy for smaller boards. Our conclusion is that the top player always has the winning strategy. By working out the boards with a height of 3 by hand, we were able to verify that our program was running exactly how it should be running and; hence, that our calculations and data were correct.

What we may undergo in the future would be the enhancement of our adaptive learning program. Although it functions correctly, the efficiency through which it finds winning strategies deteriorates while attempting to find them for larger game board sizes. Another task that may be improved is the computer's making of random moves until it stumbles upon a winning strategy. Instead, we would like to tweak the program such that the computer always chooses to capture opponent tokens when given the opportunity. Although this might alter which player possesses the winning strategies, comparing results with previous programs would help enhance our knowledge regarding who has the winning strategies and what those strategies are. Any discrepancies in the data can be further analyzed. Because our program collects all of its data, we would be allowed to analyze and depict who has the winning strategies on larger scales, giving us a deeper understanding of who has the winning strategy and how to execute the optimal strategy on larger game board sizes.

Table 1: Rock Paper Scissors Hex

Game Board	Average # of Moves To Win	Winning Strategy
3x3	7.14	Top
3x4	7.96	Top
3x5	8.98	Top
3x6	9.52	Top
4x2	9.22	Top